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ORIGINAL PAPER

Modified Gaussian Mixture Probability Hypothesis Density Filtering using Clutter Density Estimation for Multiple Target Tracking

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ABSTRACT

Gaussian mixture probability hypothesis density (GM-PHD) filtering often assumes a uniform distribution of clutter in the observation area. However, in practice, clutter is often unknown and non-uniform, necessitating accurate estimation of its spatial distribution, non-uniformity, and temporal variations. To address this problem, we proposed a modified GM-PHD filtering method with clutter density estimation for multiple target tracking. In the proposed method, first, potential target measurements within the tracking gate are eliminated to obtain the clutter measurement set. Next, the clutter density around each target is estimated. Finally, the estimated clutter density is incorporated into GM-PHD filtering, to estimate the target state and clutter density in complex clutter environments. Simulation results demonstrated that the proposed filtering method improves the performance of the GM-PHD filter in multi-target tracking scenarios with unknown clutter density.

Keywords: Gaussian mixture probability hypothesis density; Complex clutter environments; Clutter density estimation.

INTRODUCTION

Target tracking involves analyzing various observation data of moving targets detected by active and passive sensors, such as radar, infrared, laser, electronic support, and intelligence measures. Through optimization and comprehensive processing, real-time capture of targets is achieved, and the number, location, and speed of targets in the monitoring area are estimated to identify target attributes, analyze intentions, and assess a situation to facilitate threat assessment (Yang *et al.* 2023a).

Target tracking technology encompasses single-target tracking and multi-target tracking (MTT). The origins of single-target tracking technology can be traced back to the 1930s. The Kalman filter is a typical example of single-target tracking technology and exhibits excellent performance in single-sensor single-target tracking. However, with the increasing complexity of battlefield environments, single-target tracking technology struggles to meet tactical requirements. Wax (1955) proposed the concept of MTT, which was initially employed in semiautomatic ground air defense systems.

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With the development of sensor networks and information technology, MTT technology has rapidly progressed (Yang *et al.* 2023b). An MTT system continuously tracks and predicts the states of multiple targets by using measurements obtained from various sensors. MTT algorithms can be broadly categorized into two types. The first type is data association-based MTT algorithms, which operate on the principle of matching sensor-acquired measurements with targets and subsequently updating and estimating the status of each target with good correlation. However, traditional MTT algorithms, including the nearest neighbor (NN) method (Singer *et al.* 1972) and the joint probabilistic data association (JPDA) algorithm (Fortmann *et al.* 1983), rely on highly precise correlation technology; otherwise, correlation errors can lead to inaccurate estimations, greatly increasing the operational cost of the algorithm. Thus, implementing data-association algorithms in complex backgrounds is challenging. The other type includes MTT algorithms based on non-data association and random finite set (RFS) theory. Mahler (2003) proposed the probability hypothesis density (PHD) filter and provided closed-form solutions for the PHD filter under both linear Gaussian and nonlinear non-Gaussian conditions. The Gaussian mixture PHD (GM-PHD) filter (Vo and Ma 2006) and the sequential Monte Carlo PHD (SMC-PHD) filter (Clark and Vo 2007) have been developed. To achieve a more accurate estimation of the number of targets, the cardinalized PHD (CPHD) filter (Mihaylova *et al.* 2014; Pasha *et al.* 2009) has been proposed. Closed-form solutions for linear Gaussian and nonlinear Gaussian conditions have been obtained, resulting in the GM-CPHD filter (Schikora *et al.* 2013) and the SMC CPHD filter (Mahler 2009). The PHD filter framework effectively avoids the need for data association between measurements and targets, involves low computational costs, and accurately estimates the number of multiple targets and their motion states in the presence of noise and sensor omission (Beard *et al.* 2013; Huang *et al.* 2022; Li *et al.* 2018).

In cluttered environments, uncertainty exists in measurements, which makes it difficult to distinguish targets from clutter. However, most existing MTT algorithms, whether based on data association or RFS, assume a uniform clutter distribution across the observation area. In real-world scenarios, clutter is often unknown and non-uniform, requiring accurate estimation of its spatial distribution, non-uniformity, and time-varying density. To address this problem, various algorithms have been proposed. In a previous study (Lian *et al.* 2010), a comprehensive approach was presented to address the problem of estimating unknown clutter in MTT, by using the finite mixing model and the expectation maximization (EM) algorithm. However, the clutter model's sensitivity to initial values affects the estimation accuracy. Moreover, the traditional EM algorithm requires substantial computational resources and exhibits slow convergence, making it unsuitable for real-time tracking in complex cluttered backgrounds. Yan and Han (2012) proposed an algorithm based on entropy distribution to estimate unknown clutter by considering entropy distribution as the prior for mixed weights in the clutter density model, thereby enhancing the tracking performance of PHD filters in cluttered environments. However, this approach fails to meet real-time performance requirements for practical tracking. In a previous study (Chen *et al.* 2015), an MTT algorithm based on kernel density estimation was proposed for the case when the clutter density is unknown. This algorithm employs a kernel density estimator to estimate the probability density function (PDF) of clutter spatial distribution.

The main contributions of this paper are as follows:

- Unlike the existed clutter density estimation, the predicted target state information is fully utilized by our approach, which helps to identify and estimate clutters in the region surrounding each target for improving the tracking performance.
- By incorporating clutter density estimation into the update equation for MTT, the clutter interference can be reduced and the trajectory of the target can be accurately estimated.
- Due to update the prediction component with latent target measurements, much less Gaussian components are needed to be pruned and merged compared with existed approaches, which reduces the time complexity of the algorithm.

The rest of this paper is organized as follows: in the problem formulation, the multi-target RFS modeling and the GM-PHD recursion are presented, and the GM-PHD filtering approach with clutter density estimation is proposed in the modified GM-PHD with clutter density estimation; in the simulation results and discussions, simulation results are provided. Finally, conclusions are presented.

PROBLEM FORMULATION

Multi-target RFS modeling and the PHD filter

The MTT method based on RFS theory defines the states and measurements of multiple targets as RFS variables:

$$X_k = \left\{ x_{k,1}, x_{k,2}, \ldots, x_{k,N_k} \right\} \tag{1}$$

$$Z_k = \left\{ z_{k,1}, z_{k,2}, \ldots, z_{k,M_k} \right\} \tag{2}$$

where X_k is a random set representing the states of multiple targets at time k, Z_k is the measured random set at time k, N_k is the number of targets at time k, M_k is the number of measured values at time k, $x_{k,i}\left(i=1,2,\ldots,N_k\right)$ is the state of the i th goal at time k, and $z_{k,j}\left(j=1,2,\ldots,M_k\right)$ is the j th measurement at time k.

Let the multi-target state set at time k-1 be represented by x_{k-1}, then the multi-target state set at time k can be expressed as follows:

$$X_k = \left\{ \bigcup_{x_{k-1}\in X_{k-1}} B_{k|k-1}\left(x_{k-1}\right) \right\} \cup \left\{ \bigcup_{x_{k-1}\in X_{k-1}} S_{k|k-1}\left(x_{k-1}\right) \right\} \cup \Gamma_k \tag{3}$$

where $B_{k|k-1}\left(x_{k-1}\right)$ and $S_{k|k-1}\left(x_{k-1}\right)$ respectively represent the RFS of states of the x_{k-1}-derived target and the survival target at time k, and Γ_k represents the random set of new target states at time k.

At time k, each state $x_k \in X_k$ produces an RFS $\Theta_k\left(x\right)$, where $\Theta_k\left(x\right)$ is $\left\{z_k\right\}$ when the target is detected and $\Theta_k\left(x\right)$ is an empty set when the target is not detected. In the actual environment, in addition to measurements from actual targets, the sensor may receive false alarms or clutter sets K_k due to the interference of clutter.

Accordingly, the observation of multiple targets can be expressed using RFS modeling as follows:

$$Z_k = K_k \cup \left[\bigcup_{x\in X_k} \Theta_k\left(x\right) \right] \tag{4}$$

The PHD filter is an approximation of the multi-target Bayes filter, which makes the recursion computationally tractable. From the PHD function of the multi-target state RFS, the local maxima can be used to generate the state estimates of targets.

The PHD recursion is as follows:

$$v_{k|k-1}(x) = \int p_{S,k}(\zeta) f_{k|k-1}(x\,|\,\zeta) v_{k-1}(\zeta) d\zeta + \int \beta_{k|k-1}(x\,|\,\zeta) v_{k-1}(\zeta) d\zeta + \gamma_k(x) \tag{5}$$

$$v_k(x) = [1 - p_{D,k}(x)] v_{k|k-1}(x) + \sum_{z\in Z_k} \frac{p_{D,k}(x) g_k(z\,|\,x) v_{k|k-1}(x)}{k_k(z) + \int p_{D,k}(\xi) g_k(z\,|\,\xi) v_{k|k-1}(\xi) d\zeta} \tag{6}$$

where $\gamma_k(x)$ denotes the intensity of spontaneous target birth, $f_{k|k-1}(.\,|\,.)$ is the probability density of the Markov transition between target states, $\beta_{k|k-1}(.\,|\,.)$ denotes the intensity of a spawned target, $g_k(.\,|\,.)$ is the likelihood function of measurement to state, $P_{S,k}$ is the survival probability of targets and $P_{D,k}$ is the detection probability.

The GM-PHD recursion

For the linear Gaussian multi-target model, Vo and Ma (2006) provides a closed-form solution to the PHD recursion (Eqs. 5 and 6). More concisely, these propositions show how the Gaussian components of the posterior intensity are analytically propagated to the next time.

Step 1: Prediction

At time k, assume that the GM-PHD of the posterior strength for multiple targets is as follows:

$$v_{k-1}(x)=\sum_{i=1}^{J_{k-1}} w_{k-1}^{(i)} N\left(x; m_{k-1}^{(i)}, P_{K-1}^{(i)}\right) \tag{7}$$

where J_{k-1} is the number of Gaussian components at time k-1, X is the value in the target state space and $w_{k-1}^{(i)}$, $m_{k-1}^{(i)}$, and $P_{k-1}^{(i)}$ are the weight, mean, and variance of the i-th Gaussian component at time k-1, respectively.

The PHD of the target that survived from k-1 to k can be expressed as follows:

$$v_{k|k-1,S}(x)=P_S\sum_{i=1}^{J_{k-1}} w_{k-1}^{(i)} N\left(x; m_{k|k-1}^{(i)}, P_{k|k-1}^{(i)}\right) \tag{8}$$

$$m_{k|k-1}^{(i)}=Fm_{k-1}^{(i)} \tag{9}$$

$$P_{k|k-1}^{(i)}=Q+FP_{k-1}^{(i)}F^T \tag{10}$$

where P_S is the survival probability of the Gaussian component, $m_{k|k-1}^{(i)}$ and $P_{k|k-1}^{(i)}$ are respectively the mean and covariance of the i-th Gaussian survival component at the time k.

The new component at time k can be expressed as follows:

$$B_k(x)=\sum_{i=1}^{J_{B,k}} w_{B,k}^{(i)} N\left(x; m_{B,k}^{(i)}, P_{B,k}^{(i)}\right) \tag{11}$$

where $w_{B,k}^{(i)}$, $m_{B,k}^{(i)}$, and $P_{B,k}^{(i)}$ are respectively the weight, mean, and covariance of the i-th newborn component at time k, and $J_{B,k}$ is the number of newborn Gaussian components at time k.

The GM of the predicted strength at time k is the sum of the surviving Gaussian component $v_{k|k-1,s}(x)$ at time k and the newborn Gaussian component $B_k(x)$ at time k:

$$v_{k|k-1}(x)=v_{k|k-1,S}(x)+B_k(x) \tag{12}$$

Step 2: Updating

By using the estimated clutter density, the predicted Gaussian component can be updated with the potential target measurement $\hat{Z}_k^t$ by using the GM-PHD filter. This reduces the clutter component in the Gaussian component and provides posterior intensity for multiple targets:

$$v_k(x)=(1-P_d)v_{k|k-1}(x)+\sum_{z\in\hat{Z}_k^t}\frac{P_d g_k(z|x)v_{k|k-1}(x)}{c_k^i(z)+\int P_d g_k(z|\xi)v_{k|k-1}(\xi)d\xi} \tag{13}$$

where P_d is the detection probability, $v_{k|k-1}(x)$ is the prediction intensity, $g_k(\cdot)$ is the likelihood function, and $c_k(z)$ is the clutter density.

Step 3: Gaussian component trimming and merging

As time progresses, the number of Gaussian terms in the posterior PHD increases, leading to higher computational complexity. If there are $J_{k-1|k-1}$ GM at time k-1, then the number of GM items at time k can be obtained as follows:

$$J_{k|k}=\left(J_{k-1|k-1}\left(1+J_{\beta,k}\right)+J_{b,k}\right)\left(1+|Z_k|\right) \tag{14}$$

where $J_{\beta,k}$, $J_{b,k}$, and $|Z_k|$ are respectively the number of GM terms of the derived target state set, the new target state set, and the number of measurements.

To prevent an excessive increase in the number of Gaussian terms, it is necessary to discard Gaussian components with low weights. This process is known as Gaussian component clipping and involves trimming the Gaussian components $m_k^{(i)}$ in the posterior intensity function (Eq. 13) and retaining only those Gaussian components whose weights are greater than the clipping threshold T_p.

$$w_k^{(i)} > T_p, i = 1, 2, \ldots, J_k \tag{15}$$

Furthermore, to reduce the total number of Gaussian terms, multiple Gaussian components that exhibit similar distributions and meet the merging threshold T_m can be combined into a single Gaussian term.

Accordingly, Gaussian components $m_k^{(i)}$ and $m_k^{(j)}$ that satisfy the following condition are combined:

$$\left(m_k^{(i)} - m_k^{(j)}\right)^T \left(P_k^{(i)}\right)^{(-1)} \left(m_k^{(i)} - m_k^{(j)}\right) \leq T_{\mathrm{m}} \tag{16}$$

Finally, the GM term $\left\{\tilde{w}_k^{(i)}, \tilde{m}_{k|k}^{(i)}, \tilde{P}_{k|k}^{(i)}\right\}_{i=1}^{\tilde{J}_{k|k}}$ at time k is obtained.

Step 4: Extraction of multiple target states

Because the mean of each Gaussian term corresponds to a local extremum point of the posterior strength, the weights of the GM term can be used to obtain the states of multiple targets. The GM term $\left\{\tilde{w}_k^{(i)}, \tilde{m}_{k|k}^{(i)}, \tilde{P}_{k|k}^{(i)}\right\}_{i=1}^{J_{k|k}}$ of time k posterior strength obtained in step 5 is used for extracting the multi-target states.

MODIFIED GM-PHD WITH CLUTTER DENSITY ESTIMATION

In the original GM-PHD filter, it is assumed that the number of clutter occurrences follows the Poisson distribution and that the distribution is uniform over the entire range. Although this assumption works well in situations with low clutter and minimal changes, it becomes problematic in real-world scenarios where clutter distribution is unknown and varies over time. In such cases, this assumption leads to difficulties in accurately tracking targets, resulting in false alarms and decreased tracking accuracy and robustness.

Partial clutter density estimation

In this study, we aimed to address the challenge of estimating clutter density accurately in the presence of unknown and time-varying clutter distribution. We aimed to utilize measurement data received from sensors to identify and estimate clutter in the region surrounding each target. By incorporating this clutter density estimation into the update equation of the MTT, the number of targets and their states can be accurately determined.

To estimate clutter density, target measurements are separated from clutter measurements in the observed values by applying an elliptical wave gate, centered on the predicted position of the tracked target. The wave gate determines the range of the current observed value of the target; its size is determined based on the probability of correctly receiving echoes. The objective is to ensure that real measurements fall into the wave gate with a high probability while minimizing the inclusion of irrelevant values.

The i-th Gaussian component $m_{k-1}^{(i)}$ at time k-1 is predicted, and the tracking gate for the i-th predicted Gaussian component $m_{k-1}^{(i)}$ is calculated as follows:

$$J^{(j,i)} = \tilde{z}_k^{(j,i)T} \left[S_k^{(i)} \right]^{-1} \tilde{z}_k^{(j,i)} \le g^2 \tag{17}$$

$$\tilde{z}_k^{(j,i)} = z_k^{(j)} - h_k \left(m_{k|k-1}^{(i)} \right) \tag{18}$$

where g is the tracking gate parameter.

From the measurement set Z_k at time k, the NN measurement $\hat{Z}_k^t$ falling into the tracking gate is selected, and $\hat{Z}_k^t$ is recorded as the potential target measurement. Next, the current clutter measurement set $\hat{Z}_k^t$ is obtained by removing $\hat{Z}_k^t$ from the measurement set Z_k at time k.

The position of the j-th clutter is $\left(z_{x,k}^{(j)}, z_{y,k}^{(j)} \right) \left(j = 1, 2, \ldots, C \right)$.

The mean of the i-th prediction Gaussian component $m_{k|k-1}^{(i)}$ at time k is $(x_{k-1|k}^{(i)}, y_{k-1|k}^{(i)})(i = 1, 2, \ldots, J_{(k+1)} + J_{B,k})$.

The distance between each clutter and each Gaussian component of the predicted mean is calculated as follows:

$$d_{ij} = \sqrt{\left(x_{k-1|k}^{(i)} - z_{x,k}^{(j)} \right)^2 + \left(y_{k-1|k}^{(i)} - z_{y,k}^{(j)} \right)^2} \tag{19}$$

Next, the distance is sorted in ascending order.

Generally, clutter points with shorter distances have a greater influence on the surrounding density value of the target, whereas clutter points with longer distances have a smaller influence on the estimated density value. Therefore, to calculate the clutter density around each target, it is sufficient to consider the situation of the NN clutter. By incorporating the information of the NN clutter into the PDF, the clutter density value around the target can be determined.

By taking the average distance between the i-th target and each clutter as the distance threshold β, we obtain:

$$\beta = \frac{\sum_{j=1}^{C} d_{ij}}{C} \tag{20}$$

Let the number of distance sets be less than the distance threshold of β be η, and the distance be $\max\left(d_{ij} \le \beta \right)$.

Accordingly, the clutter density near the i-th predicted Gaussian component can be expressed as follows:

$$c_k^i \left(z \right) = \frac{\eta}{\pi \left(\max \left(d_{ij} \le \beta \right) \right)^2} \tag{21}$$

Clutter estimation GM-PHD (CE-GM-PHD)

We present a modified approach that combines GM-PHD filtering with clutter density estimation. By utilizing the predicted and measured values of the GM intensity for multiple targets, we estimated the clutter set. Next, we integrated the clutter density around each target with the GM-PHD filter, to estimate the state of multiple targets. The algorithm's flow table is illustrated in Table 1.

Table 1. The flowchart of the proposed filtering method.

Step	Title	Description
1	Initialize	Suppose that the function of the target set of states is a Gaussian mixture.
2	Prediction	The survival target is predicted according to the posterior strength of multiple targets at k-1 time. Predict new birth and derived targets.
3	Estimation of clutter density	Use the predicted Gaussian posterior strength to find the clutter value in the measured data. Estimate the amount of clutter around each target. Find the clutter density around each target.
4	Update	The estimated clutter density (Eq. 21) is brought into the updated equation (Eq. 13), and the predicted Gaussian posterior intensity is updated with the measured value.
5	Trimming and merging	Trim the Gaussian component $m_k^{(i)}$ and retain the Gaussian component whose weight is greater than the clipping threshold T_p. Merge Gauss items that meet the merge threshold T_p.
6	Target state extraction and number estimation	The number of targets is estimated as $\hat{N}_k = round\left(\sum_{i=1}^{J_k} \omega_k^{(i)}\right)$ The target state is estimated as the mean corresponding to the Gaussian component with the greatest weight

Source: Elaborated by the authors.

Complexity analysis

The algorithm complexity is a concept that measures the effectiveness of an algorithm, representing the basic number of operations required during the execution process as time complexity. The computational complexity of the GM-PHD algorithm is mainly concentrated in the recursive process. The GM term of the conventional GM-PHD algorithm increases continuously over time. If there are m GM terms at time k-1, n new GM terms at time k, and the measurement set contains p effective measurement sets and q clutter, the time complexity of the GM-PHD algorithm is $0((m + n)(1 + (p + q))$, while the time complexity of the CE-GM-PHD algorithm is $0((m + n)(1+(p)))$. The above will be analyzed and verified by the following experimental simulations in the simulation results and discussions.

SIMULATION RESULTS AND DISCUSSIONS

To verify the effectiveness of the proposed clutter estimation algorithm, we compared it with the GM-PHD filter in different scenarios.

Uneven clutter distribution with fixed clutter count

In scenario A, MTT in a simple clutter background was investigated. We assumed that the spatial distribution of the clutter measurement is non-uniform and remains constant over time. The true clutter count per moment was set as 50, and complex spatially distributed non-uniform clutter measurements were obtained using various uniform distribution PDFs and Gaussian PDFs. The clutter measurements follow the following PDF in space:

$$c_k\left(z\right)=\omega_c^1 U\left(\cdot\right)+\sum_{i=2}^{3}\omega_c^i N\left(\cdot,m_c^i,P_c^i\right) \tag{22}$$

where the weights are $\omega_c^1 = 0.2$, $\omega_c^2 = 0.3$ and $\omega_c^3 = 0.5$, $U(\cdot) = 1/S$, where S represents the area of the monitoring area on the two-dimensional measurement space m_c^i and P_c^i are the mean and covariance matrices of Gaussian distributions, respectively.

The detection area was $[-1000,1000]\times[-1000,1000]m^2$. There were three targets in the scene, and each target moved in a straight line at a nearly uniform speed. The initial states of the targets are presented in Table 2.

Table 2. The setting of target tracking scenario A.

Target number	Initial position/m	Initial velocity/(m/s)
1	(250,250)	(2.5, -11.5)
2	(-250,250)	(11.5, -2.5)

Source: Elaborated by the authors

The parameters of the new component are presented in Table 3.

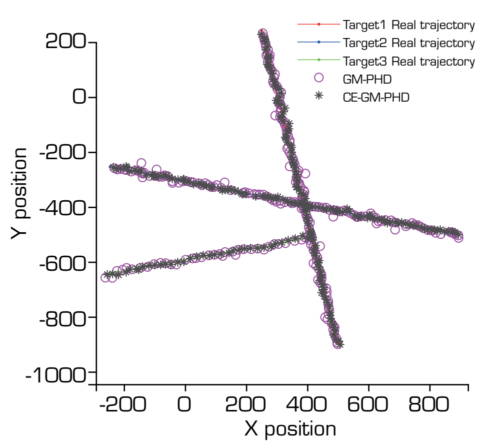
Table 3. New component parameter settings.

Component numbering	Weight	Mean/m	Variance /m²
1	0.1	$[250,0,250,0]^T$	*diag* (100,100,25,25)
2	0.1	$[-250,0,250,0]^T$	*diag* (100,100,25,25)
3	0.1	$[-250,0,-750,0]^T$	*diag* (100,100,400,400)

Source: Elaborated by the authors.

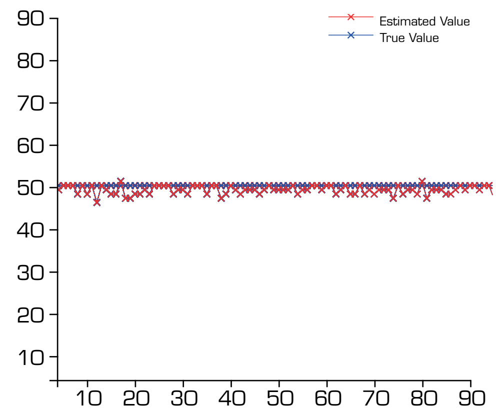
The simulation results are shown in Figs. 1–4.

As can be seen in Fig. 1, which depicts the MTT effect, there are three targets in the monitoring area, and target 1 and target 2 emerged simultaneously but occupied different positions and moved uniformly in a linear manner. Prior to and after 53 s, target 1 and target 2 first approached each other and then moved apart. At 66 s, target 3 emerged from target 1. By utilizing modified GM-PHD filtering with clutter density estimation, multiple targets can be continuously tracked throughout the simulation cycle. In contrast, the GM-PHD algorithm with fixed clutter density exhibits estimation deviations and losses of targets due to the influence of clutter distribution transformation. Experiments simulated by 100 Monte Carlo runs demonstrated that our algorithm may experience tracking bias during certain periods with simple clutter distribution. However, with continuous tracking filtering, the tracking bias returns to a smaller value, eliminating any target losses. In other words, the CE-GM-PHD algorithm demonstrates superior tracking robustness compared to the GM-PHD algorithm with fixed clutter density. A comparison between the number of clutters estimated by the CE-GM-PHD algorithm and the actual number of clutters is shown in Fig. 2.



Source: Elaborated by the authors.

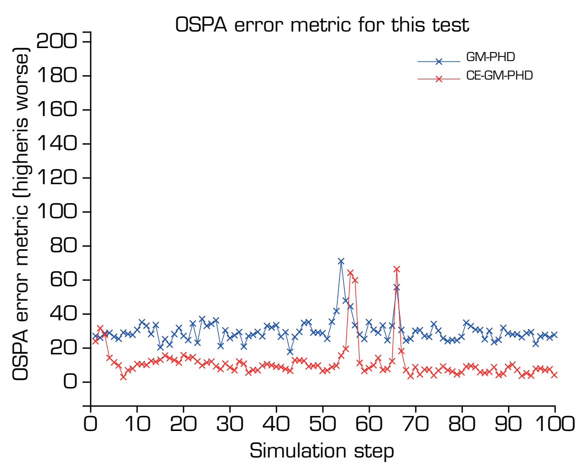
Figure 1. Multi-target tracking trajectories of two approaches.



Source: Elaborated by the authors.

Figure 2. Clutter number comparison.

To ensure an accurate comparison of the tracking performance between the two approaches, we employed the optimal subpattern assignment (OSPA) distance [Schuhmacher *et al.* 2008) as a metric to measure their tracking errors, as depicted in Fig. 3. Table 4 shows single run time of two approaches.



Source: Elaborated by the authors.

Figure 3. Tracking errors of the two approaches.

Table 4. Averaged computation time (seconds) for one run (100 steps) of two approaches in scenario A.

GM-PHD	CE-GM-PHD
67.5694	46.6473

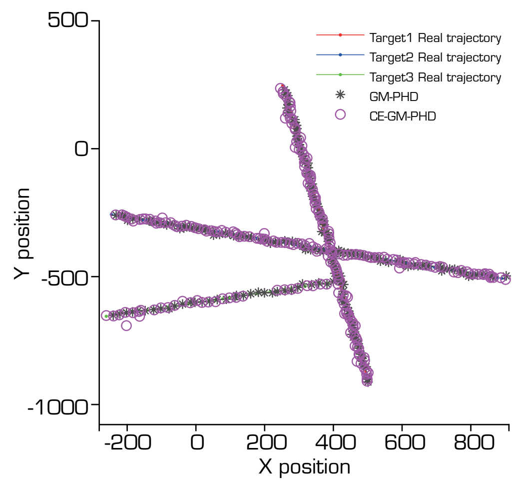
Source: Elaborated by the authors.

As can be seen in Fig. 3, the modified GM-PHD filtering algorithm with clutter density estimation outperformed the standard GM-PHD algorithm when the targets moved uniformly in a straight line without mutual interference, exhibiting superior tracking with smaller errors. However, large errors occurred when targets intersected or separated due to the influence of neighboring targets, resulting in an increased intensity of target measurements. This demonstrates the CE-GM-PHD algorithm's ability to handle the complex relationship between clutter density and target distribution in complex environments and its ability to accurately estimate the clutter density surrounding each target. Furthermore, the CE-GM-PHD algorithm provided a more precise approximation of the actual clutter distribution, thus demonstrating its adaptability, stability, and robustness in tracking. By accurately estimating clutter intensity, it effectively reduces the computational load and enhances real-time tracking performance by eliminating the need to address inaccurate clutter Gaussian components during pruning and merging processes.

Uneven clutter distribution and variable clutter count

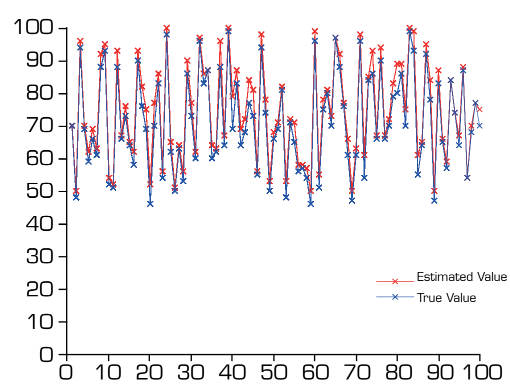
In scenario B, MTT under complex clutter backgrounds was investigated. A computer simulation was performed to randomly generate clutter instances, surpassing the number of clutter instances in scenario A. The clutter distribution in this environment was unknown. Three targets within the region exhibited uniform straight-line motion, and the same motion model parameters and initialization parameters were employed as in scenario A. The experimental results are shown in Figs. 4–6.

As can be seen from Fig. 4, under complex backgrounds, the GM-PHD algorithm with fixed clutter density was affected by clutter fluctuations, leading to deviations in target state estimation in the majority of cases. For instance, the third derived target remained untracked for a certain duration, resulting in its loss. In contrast, the CE-GM-PHD algorithm exhibited minor deviations at specific points, such as target encounters and spawns. These findings highlight the superiority of the CE-GM-PHD algorithm in handling complex clutter scenarios because it maintains accurate tracking with minimal errors and avoids target loss. The CE-GM-PHD algorithm's capability to accurately estimate the number of clutter instances at each moment is shown in Fig. 5.



Source: Elaborated by the authors.

Figure 4. Multi-target tracking trajectories of two approaches.

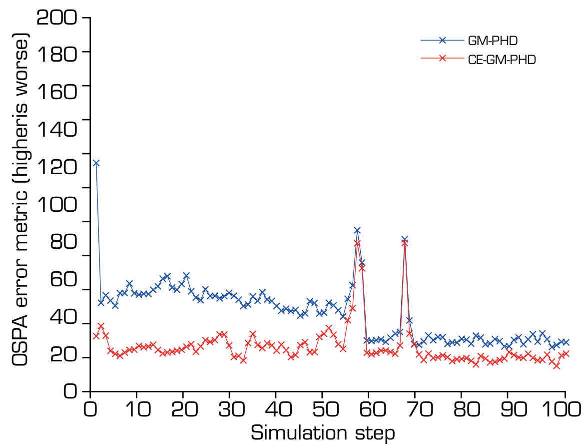


Source: Elaborated by the authors.

Figure 5. Clutter number comparison.

The tracking errors of both approaches are shown in Fig. 6. Table 5 shows single run time of two approaches.

As can be seen from Fig. 6, the CE-GM-PHD algorithm consistently and accurately tracked the upper target with minimal error compared to the GM-PHD algorithm with fixed clutter density, thus demonstrating that the CE-GM-PHD algorithm can effectively handle the complex relationship between clutter and targets in complex environments and the algorithm's ability to estimate the clutter distribution more closely to the actual distribution with strong adaptability. Moreover, the CE-GM-PHD algorithm ensures tracking stability and robustness.



Source: Elaborated by the authors.

Figure 6. Tracking errors of the two approaches.

Table 5. Averaged computation time (seconds) for one run (100 steps) of two approaches in scenario B.

GM-PHD	CE-GM-PHD
81.5346	67.7185

Source: Elaborated by the authors.

Tracking of multiple maneuvering targets

In scenario C, the tracking of multiple maneuvering targets against complex backgrounds was investigated. Clutter environments in real scenarios were simulated, and a random distribution of clutter instances was generated. The scene comprised three targets, each performing a turning movement. The initial states of the targets are outlined in Table 6.

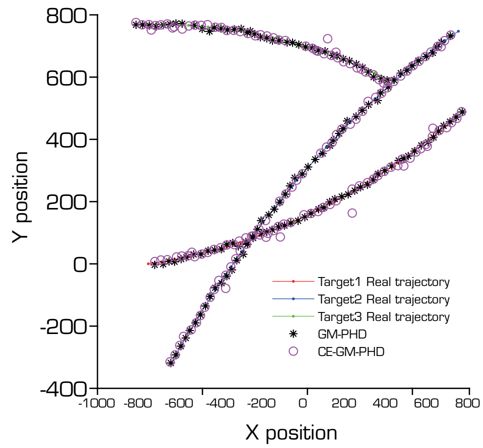
Table 6. The setting of target tracking scenario C.

Target number	Initial position/m	Initial velocity/(m/s)
1	(-750,0)	(32, 3)
2	(750,750)	(-32, -15)
3	(436.9.586.2)	(-30, 10)

Source: Elaborated by the authors.

The remaining unmentioned parameters were consistent with simulation 1. The simulation effect is shown in Figs. 7 and 8. Table 7 shows single run time of two approaches.

The multi-target trajectory results of a single simulation are shown in Fig. 7. When the target crossed and derivatives occurred, GM-PHD exhibited multiple target losses. However, the target estimation of CE-GM-PHD covered the real trajectory at every moment, thus indicating that the proposed algorithm can track multiple maneuvering targets in an unknown clutter background. Furthermore, it outperformed GM-PHD with fixed clutter density. In this simulation, both algorithms exhibited target deviation; however, the improved algorithm exhibited considerably higher tracking accuracy than GM-PHD. GM-PHD lost the target's actual location; in contrast, CE-GM-PHD excelled in tracking the target. The improved algorithm's superior performance can be attributed to the online estimation algorithm for clutter intensity.



Source: Elaborated by the authors.

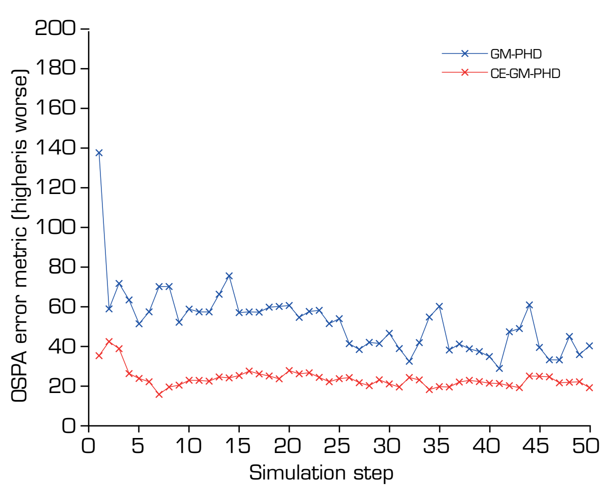
Figure 7. Multi-target tracking trajectories of two approaches.

Table 7. Averaged computation time (seconds) for one run (100 steps) of two approaches in scenario C.

GM-PHD	CE-GM-PHD
45.3684	35.2786

Source: Elaborated by the authors.

The distribution distance of the optimized sub mode representing the tracking error of multiple targets is shown in Fig. 8. CE-GM-PHD exhibited higher accuracy in tracking multiple maneuvering targets compared to GM-PHD and effectively handled the relationship between clutter and targets, resulting in clutter estimation that closely matched the real distribution. Moreover, CE-GM-PHD exhibited better stability and robustness. These findings further validate the effectiveness of the improved algorithm.

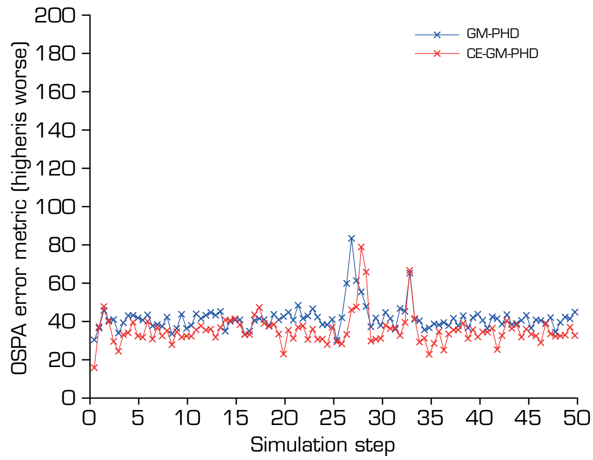


Source: Elaborated by the authors.

Figure 8. Tracking errors of the two approaches.

Clutter with non-Gaussian distribution

To verify the effectiveness of our method, we have added an MTT scenario D in which the clutter spatial distribution is a non-Gaussian distribution. We considered using the weighted mixture of different distributions (several Gaussian distributions and uniform distributions with different density) to approach practical clutter environments as possible. The comparison results between CE-GM-PHD and GM-PHD are evaluated by OSPA and shown in Fig. 9. Table 8 shows single run time of two approaches.



Source: Elaborated by the authors.

Figure 9. Tracking errors of the two algorithms.

Table 8. Averaged computation time (seconds) for one run (100 steps) of two approaches.

Gaussian mixture	CE-GM-PHD
68.4218	45.6474

Source: Elaborated by the authors.

According to above experimental results, it can be seen that when the clutter space is non-Gaussian distribution, our approach has better performance and seems more suitable for practical tracking scenarios with complex clutters. In addition, our approach has other benefits:

- There is no need to set the initial value and fit the data. The calculation is simple, which facilitates the practical tracking scenarios.
- It can estimate the clutter distribution accurately and rapidly to handle cases of time-varying and complex clutters.
- The range of the target area is adjusted adaptively, which avoids falling into the local optimal solution and improves the tracking performance.

In summary, the proposed CE-GM-PHD algorithm greatly enhances MTT performance. The simulation results demonstrated that the regional clutter estimation-based method improves the algorithm's performance without requiring clutter estimation or finding suitable initial values. In addition, it addresses the challenges posed by rapidly changing clutter scenes and accurately estimates clutter distribution in the adjacent regions of each target. The Gaussian component pruning merging method eliminates the need to handle inaccurate clutter components, resulting in simpler calculations. Consequently, the improved algorithm enables continuous real-time tracking of multiple targets in complex clutter scenarios.

CONCLUSIONS

To solve the problem that fixed clutter density affects the estimation performance of GM-PHD filtering, in this paper, we proposed the CE-GM-PHD filtering method. The proposed method performs clutter density estimation to improve filtering accuracy. The CE-GM-PHD filtering method performs potential target measurements and clutter measurements through a combination of multi-target GM intensity predictions and measurements. Furthermore, it estimates the clutter density surrounding each target and incorporates it into the GM-PHD filter. The proposed filtering method achieves continuous and effective tracking by updating the predicted Gaussian component with potential target measurements and jointly estimating the state of multiple targets.

CE-GM-PHD does not rely on the prior clutter distribution and can effectively estimate the clutter distribution, thus making it highly suitable for complex scenes. The proposed filtering method not only enhances real-time tracking performance, but also improves the accuracy of target tracking. In future research, we will explore the application of CE-GM-PHD in extended MTT.

AUTHORS' CONTRIBUTION

Conceptualization: Lifan Sun; **Data curation:** Lifan Sun and Wenhui Xue; **Formal analysis:** Lifan Sun; **Acquisition of funding:** Lifan Sun; **Research:** Lifan Sun and Wenhui Xue; **Methodology:** Lifan Sun; **Project administration:** Wenhui Xue and Dan Gao; **Resources:** Lifan Sun and Dan Gao; **Software:** Lifan Sun and Wenhui Xue; **Supervision:** Lifan Sun and Dan Gao; **Validation:** Lifan Sun and Wenhui Xue; **Visualization:** -; **Writing - Preparation of original draft:** Lifan Sun and Wenhui Xue; **Writing - Proofreading and editing:** Lifan Sun and Wenhui Xue.

CONFLICT OF INTEREST

Nothing to declare.

DATA AVAILABILITY STATEMENT

All dataset were generated or analyzed in the current study.

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REFERENCES

Beard M, Vo BT, Vo BN, Arulampalam S (2013) A partially uniform target birth model for Gaussian Mixture PHD/CPHD filtering. IEEE Trans Aerosp Electron Syst IEEE T Aero Elec Sys 49(4):2835-2844. https://doi.org/10.1109/TAES.2013.6621859

Chen X, Tharmarasa R, Kirubarajan T, McDonald M (2015) Online clutter estimation using a Gaussian kernel density estimator for multitarget tracking. IET Radar Sonar Navig 9(1):1-9. https://doi.org/10.1049/iet-rsn.2014.0037

Clark D, Vo BN (2007) Convergence analysis of the Gaussian mixture PHD filter. IEEE Trans Signal Process 55(4):1204-1212. https://doi.org/10.1109/TSP.2006.888886

Fortmann T, Bar-Shalom Y, Scheffe M (1983) Sonar tracking of multiple targets using joint probabilistic data association. IEE J Ocean Eng 8(3):173-184. https://doi.org/10.1109/JOE.1983.1145560

Huang Q, Xie L, Su H (2022) Estimations of time-varying birth cardinality distribution and birth intensity in Gaussian mixture CPHD filter for multi-target tracking. Signal Process 190:108321. https://doi.org/10.1016/j.sigpro.2021.108321

Li C, Wang W, Kirubarajan T, Lei P (2018) PHD and CPHD filtering with unknown detection probability. IEEE Trans Signal Process 66(14):3784-3798. https://doi.org/10.1109/TSP.2018.2835398

Lian F, Han C, Liu W (2010) Estimating unknown clutter intensity for PHD filter. IEEE Trans Aerosp Electron Syst 46(4):2066-2078. https://doi.org/10.1109/TAES.2010.5595616

Mahler R (2009) CPHD and PHD filters for unknown backgrounds I: Dynamic data clustering. Sensors and systems for space applications III. SPIE 7330:140-151. http://doi.org/10.1117/12.818022

Mahler RPS (2003) Multitarget Bayes filtering via first-order multitarget moments. IEEE Trans Aerosp Electron Syst 39(4):1152-1178. https://doi.org/10.1109/TAES.2003.1261119

Mihaylova L, Carmi AY, Septier F, Gning A, Pang SK, Godsill S (2014) Overview of Bayesian sequential Monte Carlo methods for group and extended object tracking. Digital Signal Process 25:1-16. https://doi.org/10.1016/j.dsp.2013.11.006

Pasha SA, Vo BN, Tuan HD, Ma WK (2009) A Gaussian mixture PHD filter for jump Markov system models. IEEE Trans Aerosp Electron Syst 45(3):919-936. https://doi.org/10.1109/TAES.2009.5259174

Schikora M, Koch W, Streit R, Cremers D (2013) A sequential Monte Carlo method for multi-target tracking with the intensity filter. Advances in intelligent signal processing and data mining: theory and applications. Berlin: Springer.

Schuhmacher D, Vo BT, Vo BN (2008) A consistent metric for performance evaluation of multi-object filters. IEEE Trans Signal Process 56(8):3447-3457. https://doi.org/10.1109/TSP.2008.920469

Singer RA, Sea RG, Housewright KB (1972) Comparison of theoretical and simulated performance of optimal and suboptimal filters in a dense multitarget environment. Paper presented 1972 IEEE Conference on Decision and Control and 11th Symposium on Adaptive Processes. IEEE; New Orleans, USA. https://doi.org/10.1109/CDC.1972.269106

Vo BN, Ma WK (2006) The Gaussian mixture probability hypothesis density filter. IEEE Trans Signal Process 54(11):4091-4104. https://doi.org/10.1109/TSP.2006.881190

Wax N (1955) Signal-to-noise improvement and the statistics of track populations. J Appl Phys 26(5):586-595. https://doi.org/10.1063/1.1722046

Yan XX, Han CZ (2012) Multi-target tracking algorithm based on online clutter intensity estimation. J Control Decis 27(4):507-512.

Yang C, Cao X, Shi Z (2023a) Road-map aided Gaussian mixture labeled multi-Bernoulli filter for ground multi-target tracking. IEEE Trans Veh Technol 72(6):7137-7147. https://doi.org/10.1109/TVT.2023.3240740

Yang J, Xu M, Li F, Yang L (2023b) Adaptive tracking and classification algorithm for multiple extended targets based on irregular shape driven. Digital Signal Process 136:103992. https://doi.org/10.1016/j.dsp.2023.103992