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Determination of skin friction on a rotating sphere in magnetic levitation

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A simple and inexpensive experimental procedure is proposed to investigate the skin friction caused by air in a rotating sphere. A device using a magnetic field keeps the sphere levitating. The sphere is briefly boosted and begins to rotate, but the air resistance acts in opposition to the motion and consequently the period of rotation increases. The experimental data demonstrate that the rotation period of the sphere increases exponentially with time. To account for this behavior, a physical model was developed, which provides an accurate description of the data. A coefficient, similar to the drag coefficient in the Drag Equation, was used to measure the air resistance on the rotating sphere. It is denominated as skin friction coefficient, its value was determined and it is remarkable that such a simple experiment provided a result so close to similar parameter value found in the literature.
Keywords: Skin friction coefficient, drag force coefficient, sphere drag torque.

1. Introduction

The skin friction of a fluid is defined as the part of the drag force that causes shear forces on the surface of the object moving through the fluid. Skin friction acts parallel to all the small pieces on the surface, causing a braking effect (torque against rotation movement). It is influenced by the smoothness of the surface of the object, fluid viscosity and the velocity of the object relative to the fluid. Another part of drag force is the pressure drag, which is caused by collision of fluid molecules with object's surface. It depends on the size and shape of the object, the properties of the fluid and the velocity of the object relative to the fluid. In summary, the drag force is a sum of skin friction and pressure drag, and the resultant effect is to slow the object motion.

Even today, it is usually necessary to use sophisticated and expensive devices to do experimental and theoretical investigations about the aerodynamic drag force acting on different objects. For instance, some studies have focused on an airfoil [1] while others have concentrated on a sphere [2]. A conventional experimental technique involves utilizing a wind tunnel with additional apparatus to suspend or pin down the objects [3]. Such methods have limitations: the presence of attachments can affect the accuracy of experimental measurements, and differentiating skin friction from the total air drag force is challenging.

Therefore, finding new experimental procedures that allow improving the results already obtained is important. This paper presents a new method for studying air friction on a rotating sphere. Sawatzki et al., in 1970, investigated this phenomenon using a sphere mechanically attached to a support that kept it rotating at constant speed [4]. Using electronic devices, he studied the flow of air near the surface of the sphere. Our article proposes a new approach for studying the skin friction on a sphere caused by air resistance. A simple magnetic apparatus is used to levitate a slim spherical shell in the air, and the time of revolution at different times is easily recorded. The effect of air resistance is to increase the period of rotation T of the sphere over time t. The magnetic levitation of a rotating sphere is a simple and inexpensive procedure that can yield results similar to those obtained by Sawatzki. Furthermore, this method can be further refined for future studies.

The mathematical description used here is based on the translational motion of the sphere and is much simpler than that used by Dennis et al. in 1980 [5]. We propose that air resistance to rotation is either *(i)* proportional to the velocity of the rotation, or *(ii)* proportional to the square of the velocity of rotation. Since the rotational motion of the sphere has no pressure drag, the air resistance is solely due to skin friction. By analyzing the experimental data and using physical models, a value for the skin friction coefficient is obtained. The result is surprisingly accurate, as indicated by the comparison of this value with literature's values of air drag force coefficients.

2. Experimental Apparatus and Procedure

Nowadays, levitating spheres have become a common tool in various experiments [6]. The levitation of the

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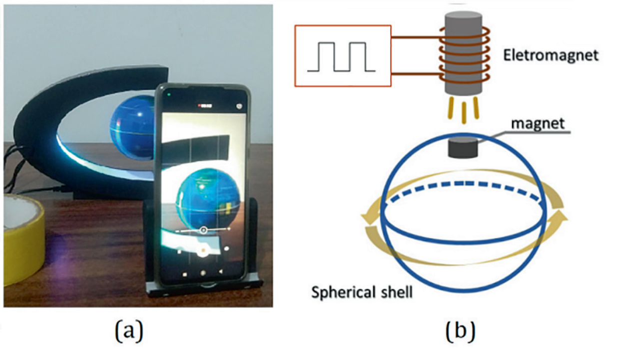

Figure 1: Experimental apparatus – rotary floating sphere in the air. (a) Picture of apparatus. (b) Schematic representation (not to scale). The resistance air force is only caused by skin friction, there is no contribution of pressure drag as in the translation motion.

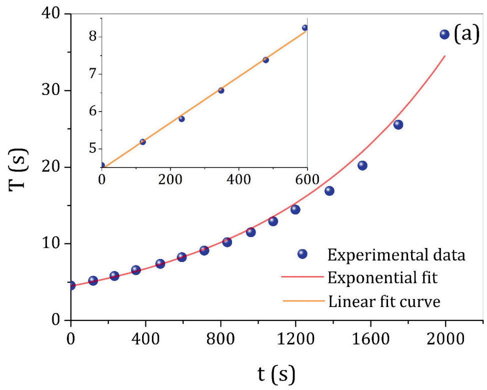

Figure 2: The graph shows the experimental data (circles) and two fitted curves (line) for the relationship between the period of rotation T of the levitating sphere and the elapsed time t. In general, points obey $T \propto exp(at)$ behavior, where $a > 0$ is a constant. The inset illustrates that the linear function, $T \propto t$, is a special case as $t \to 0$.

sphere eliminates any friction resulting from physical contact between solid parts. In Ref. [7] there is a detailed description of a simple experimental setup used in this article. Figure 1 shows the equipment used, which is a commercial product: *C SHAPE FLOATING GLOBE*. It has a small plastic spherical shell with a magnet inside. The spherical shell has mass of $M = 38.10^{-3}\ kg$, radius of $R = 44.10^{-3}\ m$ and it has a smooth surface without roughness. Henceforth, for simplicity, we will refer to the spherical shell as sphere. The small magnet inside it has the shape of a cylinder, a mass of $m = 20.10^{-3}\ kg$ and radius of $r = 80.10^{-4}\ m$ (the experimental values were obtained with an accuracy of two significant digits). The support stand is equipped with an electromagnet, a sensor, and an electronic device that generates a pulsed magnetic force, causing the spherical shell to levitate in the air.

To facilitate experimental measurement, an observation point is marked on the surface of the sphere and a small jet of air is used to set it into rotation. The air jet is stopped, and we wait until the rotational motion of the sphere becomes stable. The period of rotation T is recorded at different times t. To minimize experimental error, the period of rotation T was obtained as the average value over a time interval of Δt corresponding to 5 complete rotation cycles (while it is possible to use a method like Tracker Video analysis, theoretical analysis indicates that the method employed in this article is sufficient). Eventually, after about 60 minutes, the sphere stops spinning. As the spherical shell is made of plastic, the magnet is small and as we collect data at low speed, we disregard the effect of induced eddy currents.

Figure 2 shows the graph from experimental data, which were fitted to an exponential growth function: $T(t) = 4.5 \exp(10.10^{-4} t); R^2 = 0.995$. It is worth noting that, for small values of t, the experimental data can be approximated by a linear function, as shown in the inset and explained in the text.

3. Theoretical Analysis

An innovative mathematical analysis was used to construct the function $T(t)$ to describe these experimental data. Since the experiment involves the rotation of a sphere in the air, it must be described using spherical coordinates and physical quantities of rotation. Instead of force, we will use torque to represent the resultant effect of air resistance on the sphere's rotational motion. And instead of the relative velocity between the sphere and air, we will use the rotation velocity. To describe the rotation of the sphere under the influence of air resistance, two physical models are proposed, similar to the models used to describe the translational motion of a sphere in the air. One of the models yields an *exponential relationship* between T and t, while the other results in a *linear relationship* between these two variables.

3.1. Exponential relationship

In the case of translational motion, for low speeds, the drag force can be described by the simple equation $F = -c_1 v_{rel}$, where c_1 is a constant and v_{rel} represents the relative velocity between the object and the fluid. Similarly, for rotational sphere motion, air skin friction can also be described by the simple relationship between the torque and the rotational (angular) velocity:

$$\tau = -a_1 \omega. \tag{1}$$

The torque τ represents the skin friction acting on the entire surface of the sphere, $a_1 > 0$ is a constant of proportionality that depends on the surface smoothness of the sphere, the viscosity of the air and ω is the rotational velocity. The negative signal indicates the torque decreases the rotational velocity. Newton's second law for rotation is $\tau = I\alpha$: I is the total moment of

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inertia of the spherical shell and small cylindrical magnet inside it, $\alpha = d\omega/dt$ is the angular acceleration.

$$I\frac{d\omega}{dt} = -a_1\omega \rightarrow \frac{d\omega}{dt} = -\frac{a_1}{I}\omega. \quad (2)$$

Solving for $\omega(t)$:

$$\int_{\omega_0}^{\omega(t)} \frac{d\tilde{\omega}}{\tilde{\omega}} = \int_{t_0}^{t} -\frac{a_1}{I}dt \rightarrow \omega(t) = \omega_0 \exp\left[-\frac{a_1}{I}(t-t_0)\right], \quad (3)$$

where ω_0 is the rotational velocity in t_0. It is known that $\omega(t) = 2\pi/T$, where T is the period of rotation at t instant. Similarly $\omega_0 = 2\pi/T_0$ and, without loss of generality, assuming $t_0 = 0$, the Eq. (3) results in the following expression:

$$\frac{2\pi}{T} = \frac{2\pi}{T_0}\exp\left[\frac{-a_1}{I}(t-0)\right] \rightarrow \frac{T}{T_0} = \exp\left[\frac{+a_1}{I}t\right]. \quad (4)$$

Therefore the proposed model $\tau \propto \omega$, for low speeds, yields exponential behavior $T \propto exp(at)$, where the constant $a > 0$ incorporates the physical properties of the sphere and air.

3.2. Linear relationship

Also in the translational motion, for high speeds, the drag force can be described by a quadratic dependence between force and velocity. It is given by $F = -c_2 v_{rel}^2$, where c_2 is a new constant. Also, the skin friction in the rotation can be described in the same new way:

$$\tau = -a_2\omega^2. \quad (5)$$

The physical quantities τ and ω have the same definitions as in Eq. (1) but with different constant $a_2 > 0$. Following the same procedure as described earlier, we obtain:

$$I\frac{d\omega}{dt} = -a_2\omega^2 \rightarrow \frac{d\omega}{dt} = -\frac{a_2}{I}\omega^2. \quad (6)$$

Again, the negative signal indicates the torque decreases the rotational velocity. Solving for $\omega(t)$

$$\int_{\omega_0}^{\omega(t)} \frac{d\tilde{\omega}}{\tilde{\omega}^2} = \int_{t_0}^{t} -\frac{a_2}{I}dt \rightarrow \left(\frac{1}{\omega_0} - \frac{1}{\omega}\right) = -\frac{a_2}{I}(t-t_0). \quad (7)$$

Using the same procedure as as described previously, $\omega = 2\pi/T$, $\omega_0 = 2\pi/T_0$ and $t_0 = 0$:

$$\left[\frac{T_0}{2\pi} - \frac{T}{2\pi}\right] = -\frac{a_2}{I}t \rightarrow T = T_0 + \frac{2\pi a_2}{I}t. \quad (8)$$

Thus the proposed model $\tau \propto \omega^2$, for high speeds, yields linear behavior $T \propto \tilde{a}t$, where the constant $\tilde{a} > 0$ incorporates physical properties of the sphere and air.

4. Discussion

4.1. Similarities and differences between translational and rotational motion

The effect of air resistance on a pure sphere's rotational motion can be analyzed using a perspective similar to that of pure translational motion. In the case of pure translation, the relative velocity v_{rel} between the sphere's center of mass and the air is the speed parameter. In pure rotational motion, the speed parameter is the rotational velocity ω. For a sphere of radius R, the velocity in its surface is $v = \omega R$. The speed parameters of both translational and rotational motion are related because, for a small piece of sphere surface, is impossible to distinguish a rotational from a translational motion. Thus, for this small piece of surface, a layer of air is scraping the surface with a velocity $v_{rel} = v$. This relationship between ω and v_{rel} will be utilized below.

The graph in Fig. 2 indicates that the period of rotation T increases exponentially as t increases. As a result, as t approaches infinity, T also approaches infinity, leading to ω approaching zero. The exponential function $T \propto \exp(at)$ obtained from the $\tau \propto \omega$ model, is valid for $\omega \rightarrow 0$, this is the expected behavior at low speeds. This corresponds to the translational motion, when used $F \propto v_{rel}$ because $v_{rel} \rightarrow 0$. It is interesting to note that when $v_{rel} \gg 0$, the literature affirms that the model $F \propto v_{rel}^2$ is the most appropriate to describe the translational motion in the air. So, what happens when $\omega \gg 0$ in the rotational sphere motion? The answer can be obtained by doing Taylor series expansion, with $t \rightarrow 0$, in the Eq. (4). Note that this equation comes from $\tau \propto \omega$.

$$\frac{T}{T_0} = \exp\left[\frac{a_1}{I}t\right] \rightarrow \frac{T}{T_0} \sim 1 + \frac{a_1}{I}t. \quad (9)$$

The Eq. (9) expression is similar to Eq. (8), obtained for $\tau \propto \omega^2$. So, when $\omega \gg 0$ the behavior $\tau \propto \omega^2$ can be used in rotational motion, and this is the expected behavior at high speeds. The similarity between the translational and rotational motion can be understood in this context. Initially, the sphere rotates with high speed. However, as time progresses, the air-skin friction reduces the sphere's rotation velocity, causing ω to decrease and T to increase. The high rotational velocity, where $\omega \gg 0$, is present during the initial moments. It is within this range that the relationship $T \propto t$ is accurate, as demonstrated in the inset of Figure 2. Table 1 summarizes the presented conclusions.

According to [8], the model $F \propto v_{rel}^2$ for drag force in translational motion is applicable if the relative velocity is $v_{rel} > 24m/s$, wherein the total drag force is mainly caused by the pressure component. From data fitting in the inset of Fig. 2, the model $\tau \propto \omega^2$ for air resistance in rotational motion is applicable if the period is $T < 8.0\ s$ ($\omega > 0.79\ rad/s$), which corresponds to rotation velocity

Table 1: The best mathematical description of drag force in an object's motion in air depends on the speed range. v_{rel} is the relative velocity between the sphere's center of mass and the air. ω is the rotational velocity for a sphere of radius R.

	Translational motion	Rotational motion
Low speed	$F = -c_1 v_{rel}$	$\tau = -a_1\omega \rightarrow T = T_0 \exp\left[\frac{a_1}{I}t\right]$
High speed	$F = -c_2 v_{rel}^2$	$\tau = -a_2\omega^2 \rightarrow T = T_0 + \frac{2\pi a_2}{I}t$

$v > 3.5.10^{-2}$ m/s. This velocity threshold is obtained using the expression $v = 2\pi R/T$ for the rotating sphere.

4.2. Estimating the air skin friction coefficient for a sphere in rotational motion

The data obtained from Fig. 2 enables us to determine the Reynolds number R_e and the air skin friction coefficient c_{skin} for a rotating sphere (both are dimensionless parameters). According to fluid dynamics theory [9], the Reynolds number R_e on a sphere in a translation motion can be determined as $R_e = \rho v_{rel} D/\mu$, where $D = 2R$ is the diameter of the sphere, ρ (μ) is the density (dynamic viscosity) of air and v_{rel} is the relative velocity between the sphere's center of mass and the air. An identical expression can be applied in the rotational motion, as long as v_{rel} is regarded as the rotational speed of the sphere's surface ($v_{rel} = v = 2\pi R/T$). Using the local parameters for air properties (20° 28′ 53″ S, 54° 36′ 58″ W), $\rho = 1.1$ kg/m^3, $\mu = 1.8$ kg/ms, the range of period values 4.5 $s \leq T \leq 35$ s corresponds to $300 \geq R_e \geq 40$ values for Reynolds number. Therefore we consider the experimental data from Fig. 2 as no turbulent air motion (see section 2.2 of [4]). Numerical calculus was done by Dennis in 1980 to study the steady flow due to a rotating sphere at low and moderate Reynolds numbers [5]. Here we proposed a easier mathematical procedure.

In the general case of an object moving through a fluid, the ***total*** drag force f acts to stop it and has two components: (i) the skin friction f_{skin} and (ii) the pressure drag force f_{press}. The former acts as shear force on the surface of the object, while the latter is caused by the pressure difference between the front and back of the object.

As reported in the literature [9], the ***total*** drag force f on a sphere moving through the air can be expressed using the dimensionless drag coefficient c_W and the *Drag Equation*: $f \equiv c_W \rho A v_{rel}^2/2$. Here A is the cross-sectional area of the sphere, given by $A = \pi R^2$. The value of c_W represents the combined effect of the two drag components, skin friction and pressure drag. In pure rotational motion, $f_{press} = 0$, therefore, the total drag force acting on the sphere must be solely due to f_{skin}.

Consider a sphere with radius R rotating in the air at an angular velocity of ω. The resulting air resistance force can be obtained from the *total resistance torque* τ acting on the sphere: $f_{skin} = |\tau|/R$. If $\omega \gg 0$ then $\tau \propto \omega^2$ and Eq. (5) is valid. Thus $f_{skin} = a_2\omega^2/R$ and using $\omega = v/R$ it becomes

$$f_{skin} = \frac{a_2}{R^3}v^2. \tag{10}$$

An similar expression to *Drag Equation* can be used to describe only the skin friction f_{skin} in the total sphere's surface:

$$f_{skin} = \frac{1}{2}c_{skin}\rho A_T v_{rel}^2, \tag{11}$$

where A_T is the total sphere superficial surface, $A_T = 4\pi R^2 = 4A$. The coefficient c_{skin} is distinct from the drag coefficient c_W and it is introduced to represent the air force resistance on the rotating sphere solely due to skin friction. As discussed earlier, for a small section of the sphere's surface the velocities v and v_{rel} are the same.

Therefore, for the air layer immediately above the surface of the sphere, it is quite reasonable to assume that $v \equiv v_{rel}$. Using this equivalence and combining Eq. (10) with Eq. (11), a mathematical expression is obtained for the parameter a_2: $a_2 = 2c_{skin}\rho AR^3$.

Alternatively, a second expression for the parameter a_2 can be obtained by equalizing Eq. (8) and Eq. (9), resulting in $a_2 = T_0 a_1/2\pi$. Then, these two expressions are used to determine the c_{skin} value:

$$c_{skin} = \frac{T_0 a_1}{4\pi^2 R^5 \rho} \tag{12}$$

All parameter values on the right side of Eq. (12) are known. From data fitting of the exponential curve in Fig. 2 the initial revolution period is $T_0 = 4.5$ s and $a_1/I = 1.0.10^{-3}$, where I is the total inertia moment of sphere shell and cylinder magnet inside it: $I = 2MR^2/3 + mr^2/2$. Therefore, the skin friction coefficient estimated for the smooth spherical shell in pure rotational motion, in the range $300 \geq R_e \geq 40$, is

$$c_{skin} = 0.031. \tag{13}$$

The c_{skin} value in Eq. (13) represents the effect of f_{skin} on the entire surface of the sphere in pure rotational motion. While c_W coefficient is used to represent the total drag force, which is the sum of pressure drag and skin friction, c_{skin} represents only skin friction. The value of c_W can be obtained experimentally: a smooth sphere has $c_W = 0.47$ ($R_e \geq 10^3$) [10]. This confirms that the main contribution to the drag force is the pressure drag component.

It is worth mentioning that the value $c_{skin} = 0.031$ in Eq. (13) is very close to the drag coefficient of a streamlined body: $c_w = 0.04$ ($R_e > 10^4$). This is logical since in this object, the pressure component is assumed to be quite small.

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5. Conclusions

This article propose a simple and inexpensive experimental procedure to analyze the skin friction caused by air in a rotating sphere. Using this procedure, a mathematical description of the air resistance force acting on a levitating sphere in rotational motion is done. Experimental data of the period of rotation of the sphere as function of time reveal an exponentially increasing behavior. Two physical models were considered. The first model assumes that the resistance torque is proportional to the rotation velocity, $\tau \propto \omega$, while the second model assumes that the resistance torque is proportional to the quadratic rotation velocity, $\tau \propto \omega^2$. The first model yields a period of rotation that increases exponentially with time, $T \propto \exp(at)$, and provides a good fit to the experimental data. The second model yields a period of rotation that increases linearly with time, $T \propto \hat{a}t$, and can be seen as a special case of the first model. The coefficients a and $\tilde{a}$ are related and were determined based on physical parameters. It is assumed that the air resistance in the rotational motion of the sphere is solely due to skin friction, with no contribution from pressure drag. Based on this assumption, the parameter called c_{skin} is studied. It is equivalent to the coefficient for the total drag force c_W of the *Drag Equation*, but c_{skin} is due only the tangential friction of the fluid on the surface of the sphere. The experiment described in this article yielded a value of $c_{skin} = 0.031$ for the rotational motion of the sphere in the air. It is worth mentioning that this value is very close to the drag coefficient of a streamlined body.

Therefore, the experiment described in this article is simple and efficient for studying the exclusive contribution of the air drag force on the sphere surface without the contribution from the pressure drag force.

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