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Revisiting and comparing the Carnot Cycle and the Otto Cycle

Revisitando e comparando o Ciclo de Carnot e o Ciclo Otto

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The reversibility of the Carnot Cycle makes it the most efficient thermodynamic cycle to convert energy as heat into work operating between two thermal reservoirs at different temperatures. The Otto cycle represents an idealization of the processes in the spark-ignition 4-stroke internal combustion engine operation. Some aspects of these two crucial thermodynamic cycles will be presented here in a comparative manner. This highlights why the Carnot Cycle does not offer advantages over the Otto Cycle in the operation of real thermal engines when we consider the efficiency and work done per cycle dependent on the compression ratio of the cycles.
Keywords: Compression ratio, efficiency, thermodynamic cycles.

A reversibilidade do Ciclo de Carnot faz dele o ciclo termodinâmico de máxima eficiência na trasformação de calor em trabalho a partir de dois reservatórios térmicos a temperaturas distintas. O ciclo Otto representa uma idealização dos processos que ocorrem no funcionamento dos motores de combustão interna de 4 tempos por ignição elétrica. Serão apresentados aqui alguns aspectos desses dois importantes ciclos termodinâmicos de maneira comparativa. Com isso, evidencia-se os motivos pelos quais o Ciclo de Carnot não apresenta vantagens em relação ao Ciclo Otto quando considera-se a dependência da eficiência e do trabalho líquido por ciclo com a taxa de compressão dos ciclos.
Palavras-chave: Taxa de compressão, eficiência, ciclos termodinâmicos.

1. Introduction

According to the Carnot theorem, any engine operating between two thermal reservoirs is less efficient than a thermodynamic cycle operating reversibly between the same reservoirs [1]. After being introduced to the Carnot theorem and the Carnot cycle, students often raise the question about the feasibility of a real Carnot engine for power generation. The standard answer blames the technical impediments in implementing the Carnot cycle (e.g., zero friction, perfect thermal conductivity, heat transfer through a zero temperature difference, and close to zero power due to the shortness of adiabatic transformations succeeding slow isothermal transformations). However, beyond the efficiencies with the same thermal reservoirs, further comparisons between properties of the Carnot cycle and other cycles are rarely made to answer that question.

Thermodynamic cycle efficiencies considering different shapes in p vs. V diagram have been discussed considering different aspects until recent years. Most of them addressed the topic of locating the states where the heat changes the sign, considering triangle cycles [2], elliptical cycles [3], linear-parabolic cycles [4], Sadly Cannot cycles [5], unconventional lobe [6] and even a general formalism to treat arbitrary shapes (including star-shaped and heart-shaped cycles) [7]. Other aspects beyond the efficiency are also compared for the Carnot cycle with possible modifications on it [8].

In this work, we intend to complement the answer about the feasibility of a real Carnot engine beyond the mentioned technical impediments. The complement is given by comparing the Carnot cycle with a thermodynamic cycle which idealizes the processes in a spark-ignition 4-stroke internal combustion engine – the Otto cycle [9]. We revisit some general aspects and obtain expressions for the efficiency and work per cycle considering the Carnot cycle in Sec. 2 and the Otto cycle in Sec. 3. We compare the cycles constraining three common features between them: the compression ratio in Sec. 4, the efficiency in Sec. 5 and the thermal reservoirs in Sec. 6. A concluding discussion of the findings is offered in Sec. 7.

2. The Carnot Cycle

The Carnot cycle is defined by the four processes depicted in the p vs. V diagram of Fig. 1. The transformation $a_C \rightarrow b_C$ is performed at thermodynamic equilibrium with a hot thermal reservoir, and $c_C \rightarrow d_C$ is performed at thermodynamic equilibrium with a cold thermal reservoir. This feature allows us to identify

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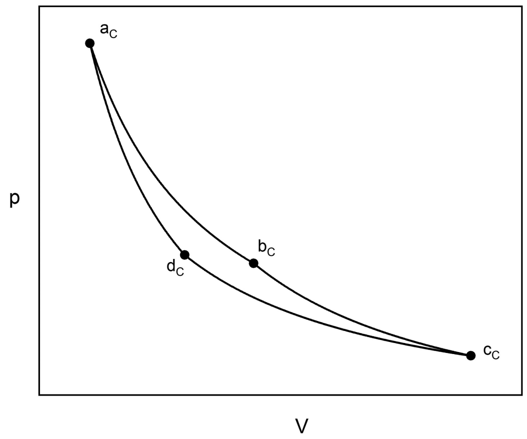

Figure 1: Schematic view of a Carnot Cycle. The processes $a_C \to b_C$ and $c_C \to d_C$ are isothermal transformations. The processes $b_C \to c_C$ and $d_C \to a_C$ are adiabatic transformations.

$a_C \to b_C$ as an isothermal expansion and $c_C \to d_C$ as an isothermal contraction. According to the First Law of Thermodynamics, as the internal energy of the gas remains unchanged during isothermal processes, the heat absorbed by the gas during $a_C \to b_C$ is integrally used to expand it. Likewise, the heat transferred from the gas to the cold reservoir during $c_C \to d_C$ comes integrally from the work done by the surroundings on the gas to contract it.

The transformations $b_C \to c_C$ and $d_C \to a_C$ occur without any contact with the thermal reservoirs and could be motivated by the inertial movement of a piston after each isothermal transformation. This feature allows us to identify $b_C \to c_C$ as an adiabatic expansion and $d_C \to a_C$ as an adiabatic contraction (without heat exchange between the system and the surroundings). According to the First Law of Thermodynamics, as there is no heat exchanged, the temperature of the gas decreases until the temperature of the cold thermal reservoir along $b_C \to c_C$. Also, it rises until the temperature of the hot thermal reservoir along $d_C \to a_C$.

The processes $a_C \to b_C$ and $c_C \to d_C$ are reversible since they are performed at equilibrium conditions between the gas and its surroundings. The processes $b_C \to c_C$ and $d_C \to a_C$ are also reversible since the entropy of the gas remains unchanged as no heat is transferred from it to the surroundings. Therefore, the whole cycle is considered reversible, meaning the universe's entropy remains unchanged after the gas completes the cycle [10, 11]. As a consequence of the Second Law of Thermodynamics, it is possible to conclude that a reversible heat engine is the most efficient one operating between two given thermal reservoirs. Remarkably, Carnot concluded that before either the First Law or the Second Law of Thermodynamics had been established [1]. Thus, the Carnot efficiency will always be greater when compared with any other thermodynamic cycle with maximum and minimum temperatures equal to that of the hot and cold thermal reservoirs used in a Carnot Cycle.

The maintenance of temperature of $a_C \to b_C$ and $c_C \to d_C$ associated with the ideal gas state equation leads to the following relations

$$p_{b_C} V_{b_C} = p_{a_C} V_{a_C}, \tag{1}$$

$$p_{d_C} V_{d_C} = p_{c_C} V_{c_C}. \tag{2}$$

The First Law of Thermodynamics associated with the ideal gas state equation leads to the following relations for the adiabatic transformations $b_C \to c_C$ and $d_C \to a_C$:

$$p_{b_C} V_{b_C}^{\gamma} = p_{c_C} V_{c_C}^{\gamma}, \tag{3}$$

$$T_{b_C} V_{b_C}^{\gamma-1} = T_{c_C} V_{c_C}^{\gamma-1}, \tag{4}$$

$$p_{d_C} V_{d_C}^{\gamma} = p_{a_C} V_{a_C}^{\gamma}, \tag{5}$$

$$T_{d_C} V_{d_C}^{\gamma-1} = T_{a_C} V_{a_C}^{\gamma-1}. \tag{6}$$

where T is the temperature γ is the adiabatic index defined by the heat capacity ratio $\gamma = c_p/c_V$. Here c_p is the heat capacity at constant pressure, and c_V is the heat capacity at constant volume. For an ideal gas $c_p = c_V + R$ (where $R = 8.314\ J.mol^{-1}.K^{-1}$ is the ideal gas constant) and $c_V = fR/2$, where f is the number of degrees of freedom (3 for a monatomic ideal gas, 5 for a diatomic ideal gas or a gas of linear molecules and 6 for a polyatomic ideal gas [12, 13]).

By dividing equation (3) by (1), it is possible to find an expression for the volume of the state b_C

$$V_{b_C}^{\gamma-1} = \frac{p_{c_C} V_{c_C}^{\gamma}}{p_{a_C} V_{a_C}}. \tag{7}$$

Similarly, by dividing equation (5) by (2), it is possible to find an expression for the volume of the state d_C

$$V_{d_C}^{\gamma-1} = \frac{p_{a_C} V_{a_C}^{\gamma}}{p_{c_C} V_{c_C}}. \tag{8}$$

We can relate the ratios V_{b_C}/V_{a_C} and V_{c_C}/V_{d_C} dividing equation (4) by (6) and recognizing that $T_{b_C} = T_{a_C} = T_h$ and $T_{d_C} = T_{c_C} = T_c$, where T_h is the hot reservoir temperature and T_c is the cold reservoir temperature. Thus, we can write

$$\frac{V_{b_C}}{V_{a_C}} = \frac{V_{c_C}}{V_{d_C}}. \tag{9}$$

It is useful to define the compression ratio of the cycle as the ratio of the largest to smallest volume of the gas. For the Carnot cycle, we have

$$r_{V_C} \equiv \frac{V_{c_C}}{V_{a_C}}. \tag{10}$$

The relevance of this parameter is rarely discussed for the Carnot cycle, while it is emphasized in the context of many other thermodynamic cycles. The compression ratio is directly related to the piston displacement during each stroke as it is proportional to the ratio of the largest to the smallest volume of the gas in the cylinder. Therefore, the relation between the Carnot cycle properties and the compression ratio is essential to investigate the technical limitations of implementing this cycle.

We use a sign convention for heat where $Q > 0$ if the heat is absorbed by the working gas from the hot reservoir and $Q < 0$ if the heat is rejected from the working gas to the cold reservoir. The heat is transferred from the hot reservoir to the working gas during the isothermal expansion $a_C \to b_C$, which leads to

$$Q_{in_C} = Q_{a_C b_C} = p_{a_C} V_{a_C} \ln\left(\frac{V_{b_C}}{V_{a_C}}\right). \quad (11)$$

Similarly, the heat is rejected from the working gas to the cold reservoir during the isothermal process $c_C \to d_C$, which leads to

$$Q_{out_C} = \mid Q_{c_C d_C} \mid = p_{c_C} V_{c_C} \ln\left(\frac{V_{c_C}}{V_{d_C}}\right). \quad (12)$$

The efficiency of a thermodynamic cycle can be defined as $e \equiv W/Q_{in}$, where W is the net work done by the gas per cycle, given by the First Law of Thermodynamics as the difference $W = Q_{in} - Q_{out}$. This leads to the relation

$$e = 1 - \frac{Q_{out}}{Q_{in}}. \quad (13)$$

Substituting equations (11) and (12) into equation (13), and using the identity expressed in equation (9) we have

$$e_C = 1 - \frac{p_{c_C} V_{c_C}}{p_{a_C} V_{a_C}}, \quad (14)$$

in which, by applying the ideal gas state equation in states c_C and a_C, it is possible to express e_C in terms of T_h and T_c as $e_C = 1 - T_c/T_h$ as it is better known.

The net work done per cycle is poorly investigated when introductory physics textbooks address the Carnot cycle. This quantity is proportional to the power performed by the engine and corresponds to the inner area of the cycle represented in a p vs. V diagram. By substituting equations (11) and (12) in $W = Q_{in} - Q_{out}$, using the identity expressed in equation (9) and the equation (7) for the volume V_{b_C} we obtain

$$W_C = \left(\frac{p_{a_C} V_{a_C}}{\gamma - 1}\right)\left(1 - \frac{p_{c_C} V_{c_C}}{p_{a_C} V_{a_C}}\right) \ln\left(\frac{p_{c_C} V_{c_C}^{\gamma}}{p_{a_C} V_{a_C}^{\gamma}}\right). \quad (15)$$

By recognizing the expression of Carnot efficiency (14) in equation (15) and by applying the definition of r_{V_C} expressed in equation (10) it is possible to write

$$\frac{W_C}{p_{a_C} V_{a_C}} = \left(\frac{e_C}{\gamma - 1}\right) \ln\left[(1 - e_C) r_{V_C}^{\gamma-1}\right]. \quad (16)$$

There is a non-monotonic dependence of W_C on e_C in the equation (16), since $W_C = 0$ for $e_C = 0$ and also for $e_C = 1 - r_{V_C}^{1-\gamma}$. The second case is surprisingly equal to the Otto cycle efficiency with the same compression ratio, as we will see in the next session.

We illustrate the non-monotonic dependence of W_C on e_C by representing some Carnot cycles of different efficiencies and the same compression ratio in the p vs. V diagrams in Fig. 2. There, we fixed $r_{V_C} = 10$ and $(V_{a_C}, p_{a_C}) = (25\ mL, 75.36\ atm)$ for all cycles. The p_{c_C} values are determined for each e_C using equation (14), while V_{b_C} and V_{d_C} are determined by equations (7) and (8), respectively. After that, the values of p_{b_C} and p_{d_C} are determined using equations (1) and (2), respectively. As we can see, the area inside the cycles grows as the efficiency becomes greater than zero until it starts to shrink as the efficiency gets close to $e_{C_{max}} = 1 - r_{V_C}^{1-\gamma} \sim 0.6$. In the case of $e_C = 0$, the volume expansion and contraction happen along the same isothermal process. In contrast, in the case of $e_C = e_{C_{max}}$, the volume expansion and contraction happen along the same adiabatic process.

3. The Otto Cycle

The Otto cycle is defined by the four processes depicted in the p vs. V diagram of Fig. 3. This cycle outlines a sequence of thermodynamic processes in a piston-cylinder system of an idealized spark-ignition 4-stroke internal combustion engine. The transformations $a_O \to b_O$ and $c_O \to d_O$ are adiabatic processes and represent the rapid expansion and compression of the gas into the cylinder by the piston movement. The transformation $d_O \to a_O$ represents the explosion of the fuel-air mixture, causing a sudden pressure increase at a constant volume. The transformation $b_O \to c_O$ represents the exhaustion of the combustion products, dropping the working gas pressure instantaneously during a constant volume process. There are also two additional processes omitted in Fig. 3: an isobaric contraction for the exhaust of wasting combustion products and an isobaric expansion at the same pressure for the intake of the cool fuel-air mixture.

The adiabatic transformations $a_O \to b_O$ and $c_O \to d_O$ obey the relations:

$$p_{b_O} V_{b_O}^{\gamma} = p_{a_O} V_{a_O}^{\gamma}, \quad (17)$$

$$T_{b_O} V_{b_O}^{\gamma-1} = T_{a_O} V_{a_O}^{\gamma-1}, \quad (18)$$

$$p_{c_O} V_{c_O}^{\gamma} = p_{d_O} V_{d_O}^{\gamma}, \quad (19)$$

$$T_{c_O} V_{c_O}^{\gamma-1} = T_{d_O} V_{d_O}^{\gamma-1}. \quad (20)$$

The compression ratio of the Otto Cycle is defined by

$$r_{V_O} \equiv \frac{V_{b_O}}{V_{a_O}} \equiv \frac{V_{c_O}}{V_{d_O}}, \quad (21)$$

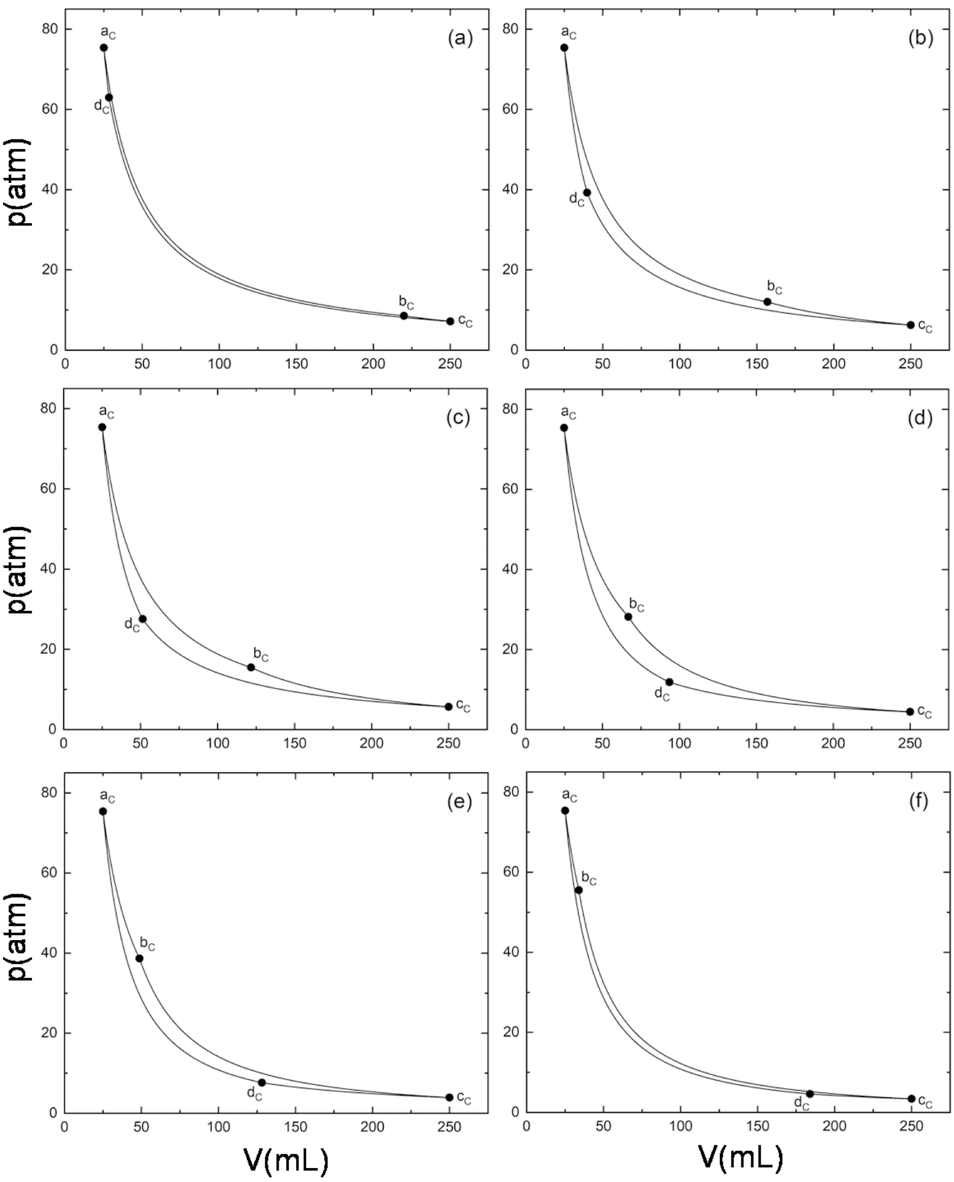

Figure 2: Examples of Carnot cycles of same compression ratio $r_{V_C} = 10$, working gas with $\gamma = 1.4$ and different efficiencies e_C. The efficiencies are (a) $e_C = 0.05$, (b) $e_C = 0.17$, (c) $e_C = 0.25$, (d) $e_C = 0.41$, (e) $e_C = 0.48$ and (f) $e_C = 0.55$.

since $V_{c_O} = V_{b_O}$ and $V_{a_O} = V_{d_O}$. It is also useful to define a pressure ratio as

$$r_{p_O} \equiv \frac{p_{a_O}}{p_{d_O}}, \tag{22}$$

where it is possible to verify that $p_{a_O}/p_{d_O} = p_{b_O}/p_{c_O}$ by dividing equation (17) by (19). The pressure ratio r_{p_O} is determined by the combustion reaction of the air-fuel mixture.

The heat is transferred to the working gas during the isochoric process $d_O \to a_O$, while the heat is rejected from the working gas during the isochoric process $b_O \to c_O$. The heat exchanged at constant volume is defined as $Q_V = nc_V \Delta T$, where n is the number of moles, which can be rewritten as $Q_V = (c_V/R)V\Delta p$ by applying the ideal gas state equation. As a result, it is possible to write

$$Q_{in_O} = Q_{d_O a_O} = \frac{c_V}{R} p_{a_O} V_{a_O} \left(1 - \frac{1}{r_{p_O}}\right), \tag{23}$$

$$Q_{out_O} = \mid Q_{b_O c_O} \mid = \frac{c_V}{R} p_{b_O} V_{b_O} \left(1 - \frac{1}{r_{p_O}}\right). \tag{24}$$

Substituting equations (23) and (24) into equation (13), we have

$$e_O = 1 - \frac{p_{b_O} V_{b_O}}{p_{a_O} V_{a_O}}, \tag{25}$$

where we can use the relation between a_O and b_O expressed by equation (17) and the compression ratio

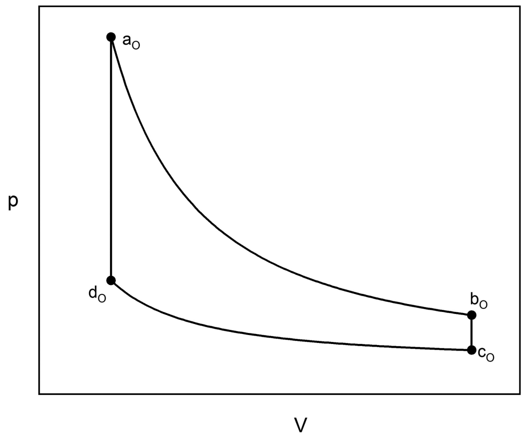

Figure 3: Schematic view of an Otto Cycle. The processes $a_O \to b_O$ and $c_O \to d_O$ are adiabatic transformations. The processes $b_O \to c_O$ and $d_O \to a_O$ are isochoric transformations.

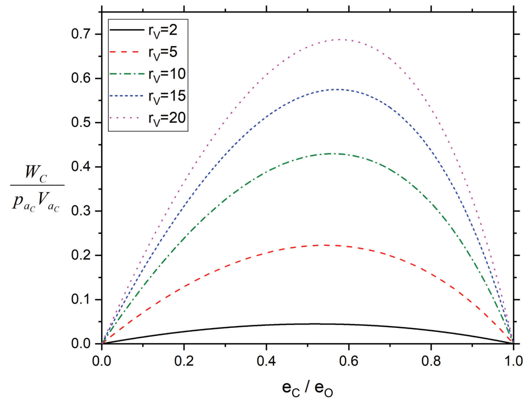

Figure 4: Work per Carnot cycles W_C in units of the highest energy state $p_{a_C} V_{a_C}$ varying with the efficiency for some values of r_V (distinguished in the legend). The working gas $\gamma = 1.4$ is the same for all cycles.

defined in (21) and write

$$e_O = 1 - \frac{1}{r_{V_O}^{\gamma-1}}. \tag{26}$$

On the other hand, we can obtain the work done per Otto cycle by substituting equations (23) and (24) into the relation $W = Q_{in} - Q_{out}$. This leads to

$$W_O = \frac{c_V}{R}\left(1 - \frac{1}{r_{pO}}\right) p_{a_O} V_{a_O}\left(1 - \frac{p_{b_O} V_{b_O}}{p_{a_O} V_{a_O}}\right). \tag{27}$$

By recognizing the expression of Otto efficiency (25) in (27) and using the relation $\gamma = c_p/c_V$, where $c_p = c_V + R$, we can write

$$\frac{W_O}{p_{a_O} V_{a_O}} = \left(\frac{e_O}{\gamma - 1}\right)\left(1 - \frac{1}{r_{pO}}\right), \tag{28}$$

which reveals a monotonic positive dependence of W_O on the parameters r_{pO} and e_O.

4. Carnot Cycle vs. Otto Cycle with $r_{V_C} = r_{V_O}$

Although the Carnot cycle is the pinnacle of efficiency when compared to any other thermodynamic cycle operating within the same limits of temperatures, it may be useful to investigate how its efficiency compares with the Otto efficiency when we constrain the compression ratio as the common parameter. By making $r_{V_C} = r_{V_O} = r_V$ on equation (16) and enforcing the argument inside the logarithm to be greater than one, we find the condition

$$e_C < 1 - \frac{1}{r_V^{\gamma-1}} \implies e_C < e_O, \tag{29}$$

i.e., the Carnot efficiency is always lesser than the Otto efficiency when both cycles have the same compression ratios.

As indicated in Fig. 2, the work per Carnot cycle presents a maximum for some efficiency e_C^* within the range $0 < e_C < e_O$. To highlight this aspect, we show in Fig. 4 the work done per Carnot cycle varying with the efficiency for some values of r_V. The efficiency is expressed in units of the Otto cycle efficiency e_O relative to each r_V while the work is expressed in units of the highest energy state $p_{a_C} V_{a_C}$. It is possible to observe that W_C is null for $e_C = 0$ and $e_C = e_O$ and the maximum work $W_{C_{max}}$ occurs for some efficiency e_C^* which depends on r_V.

The values for e_C^* can be found by making

$$\left.\frac{\partial W_C}{\partial e_C}\right|_{e_C = e_C^*} = 0, \tag{30}$$

whereby obtaining the derivative, the following expression needs to be solved

$$\ln\left[(1 - e_C^*) r_V^{\gamma-1}\right] - \frac{e_C^*}{1 - e_C^*} = 0. \tag{31}$$

The above expression cannot be analytically solved for e_C^*. The solution has to be numerically obtained, and the dependence of e_C^* on the compression ratio r_V of the Carnot cycle is depicted in Fig. 5. The same graph shows the Otto efficiencies using equation (26) to verify that e_O is greater than e_C^* for all r_V values.

Substituting the values for e_C^* in equation (16), we find the maximum work done per Carnot cycle relative to each r_V value, which is shown as the solid line of Fig. 6. To compare, we plot in the same graph the work done per Otto cycle with different pressure ratios r_{pO} using equation (28). The work values are expressed in units of the highest energy state $p_{a_C} V_{a_C}$, which is the same for all cycles. It is possible to observe that for $r_V \leq 10$ we have $W_O > W_{C_{max}}$ for all cases where $r_{pO} > 1.4$ (typical values in real spark-ignition 4-strokes internal combustion engines are $2 < r_{pO} < 4$ and $r_V \sim 10$ [9]).

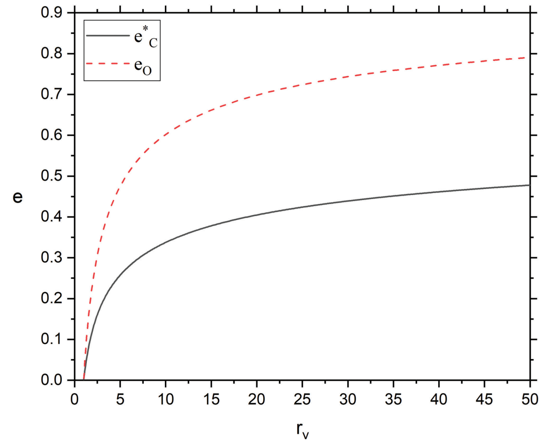

Figure 5: Carnot Cycle efficiencies e^*_C (solid line) that give a maximum work depending on r_V values considering $\gamma = 1.4$. In comparison, we depicted the Otto cycle efficiencies e_O (dashed line) relative to r_V values.

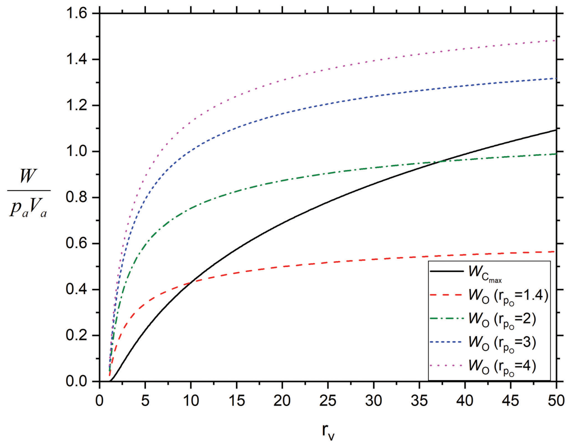

Figure 6: Maximum work per Carnot cycles $W_{C_{max}}$ (solid line) for different compression ratios r_V. In comparison, we plot the work per Otto cycles (lines distinguished in the legend) dependence on r_V for different pressure ratios r_{p_O}. Work values are expressed in units of the highest energy state $p_{a_C} V_{a_C}$. The working gas $\gamma = 1.4$ is the same for all cycles.

To illustrate an example of the Carnot cycle and the Otto cycle with the same compression ratio, we depicted both with $r_V = 10$ and the Otto cycle with $r_{p_O} = 3$ in the p vs. V diagram of Fig. 7. The Carnot cycle is at maximum work condition. The state $(V_a, p_a) = (25\ mL, 75.36\ atm)$ and the working gas $\gamma = 1.4$ are the same for both cycles. The values of connection states are described in the figure caption. According to equation (31), by considering $r_V = 10$, the maximum work value $W_{C_{max}}$ is obtained for $e^*_C = 0.337$. For the Otto cycle, equation (26) gives an efficiency of $e_O = 0.602$. The areas inside each cycle indicate that the work done per Otto cycle is greater than the maximum work done per Carnot cycle, which is corroborated by equations (16) and (28) giving $W_{C_{max}} = 82\ J$ and $W_O = 191.5\ J$.

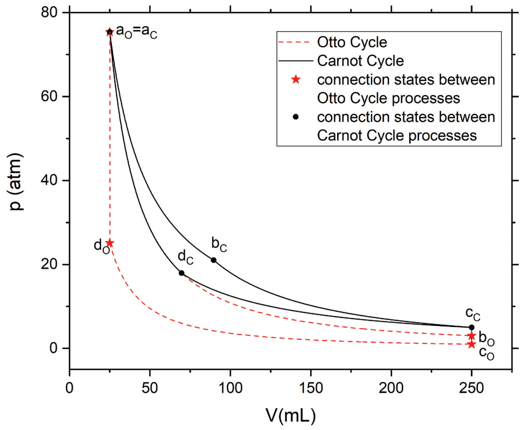

Figure 7: Carnot cycle (solid line) and Otto cycle (dashed line) for the same compression ratio $r_V = 10$ and working gas $\gamma = 1.4$. The pressure ratio for the Otto Cycle is $r_{p_O} = 3$. The Otto Cycle exhibits an efficiency of $e_O = 0.602$ and a work of $W_O = 191.5\ J$. The Carnot Cycle has an efficiency of $e^*_C = 0.337$, which gives a maximum value for the work $W_{C_{max}} = 82\ J$. For the Otto cycle, we have the connection states $(V_{b_O}, p_{b_O}) = (250\ mL, 3\ atm)$; $(V_{c_O}, p_{c_O}) = (250\ mL, 1\ atm)$ and $(V_{d_O}, p_{d_O}) = (25\ mL, 25.12\ atm)$. For the Carnot cycle we have the connection states $(V_{b_C}, p_{b_C}) = (89.64\ mL, 21.02\ atm)$; $(V_{c_C}, p_{c_C}) = (250\ mL, 5\ atm)$ and $(V_{d_C}, p_{d_C}) = (69.72\ mL, 17.93\ atm)$. The state $(V_a, p_a) = (25\ mL, 75.36\ atm)$ is chosen as being the same for both.

5. Carnot Cycle vs. Otto Cycle with $e_C = e_O$

We want to investigate the conditions for the Carnot cycle to exhibit an efficiency equal to that of the Otto cycle. Substituting e_O given by equation (26) as e_C in the equation (16) and enforcing the argument inside the logarithm to be greater than one, we find the condition

$$r_{V_C} > r_{V_O}, \tag{32}$$

i.e., the compression ratio of the Carnot cycle must be greater than that of the Otto cycle to both have the same efficiency. By equaling the right side of equations (14) and (26) we find the follow additional criteria to be obeyed

$$\frac{p_{c_C}}{p_{a_C}} = \frac{1}{r_{V_C} r_{V_O}^{\gamma-1}}. \tag{33}$$

In addition, suppose the Carnot cycle is forced to perform the same work per cycle as the Otto cycle. In this case, is possible to determine an expression for r_{V_C} by equaling the right sides of equations (16) and (28) with $e_C = e_O = 1 - r_{V_O}^{1-\gamma}$. This leads to the relation

$$r_{V_C} = r_{V_O} \exp\left[\frac{r_{p_O} - 1}{r_{p_O}(\gamma - 1)}\right]. \tag{34}$$

To illustrate an example of the Carnot cycle and the Otto cycle with the same efficiency and work per cycle,

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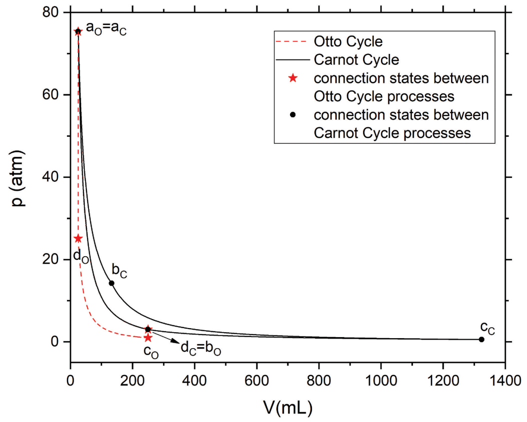

Figure 8: Carnot cycle (solid line) and Otto cycle (dashed line) for the same efficiency $e_C = e_O = 0.602$, work per cycle $W_C = W_O = 191.5\ J$ and working gas $\gamma = 1.4$. The parameters of the Otto Cycle are the same as used in Fig. 7. The Carnot Cycle must present $r_{V_C} = 52.9$ to achieve the conditions $e_C = e_O$ and $W_C = W_O$. For the Carnot cycle we have the connection states $(V_{b_C}, p_{b_C}) = (135.28\ mL, 13.93\ atm)$; $(V_{c_C}, p_{c_C}) = (1326.4\ mL, 0.57\ atm)$ and $(V_{d_C}, p_{d_C}) = (250\ mL, 3\ atm)$. The state $(V_a, p_a) = (25\ mL, 75.36\ atm)$ is chosen as being the same for both.

we depicted both in the p vs. V diagram of Fig. 8. The state $(V_a, p_a) = (25\ mL, 75.36\ atm)$ and the working gas $\gamma = 1.4$ are the same for both cycles. The Otto cycle is identical to that of Fig. 7, i.e., $r_{V_O} = 10$, $r_{p_O} = 3$ and $e_O = 0.602$. Substituting these values in equation (34), we find that the Carnot Cycle must present a compression ratio as great as $r_{V_C} = 52.9$ to achieve $e_C = e_O$ and $W_C = W_O$. By using $r_{V_C} = 52.9$ and $r_{V_O} = 10$ in equation (33) to obtain p_{c_C} and in equation (10) to obtain V_{c_C}, we find the minimum energy state as $(V_{c_C}, p_{c_C}) = (1326.4\ mL, 0.57\ atm)$.

6. Carnot Cycle vs. Otto Cycle with $\frac{T_{c_C}}{T_{a_C}} = \frac{T_{c_O}}{T_{a_O}}$

The most known comparative scenario is when the Carnot cycle is constrained to operate between the same extreme temperatures as any other thermodynamic cycle. In this case, as a consequence of the Second Law of Thermodynamics, the Carnot efficiency is inexorably the greatest. However, there is a price to pay in terms of the Carnot cycle compression ratio when we compare it with the Otto cycle with the same extreme temperatures.

Considering an ideal gas operated by both cycles, the relation $T_{c_C}/T_{a_C} = T_{c_O}/T_{a_O}$ leads to

$$\frac{p_{c_C} V_{c_C}}{p_{a_C} V_{a_C}} = \frac{p_{c_O} V_{c_O}}{p_{a_O} V_{a_O}}. \tag{35}$$

Combining the above relation with equations (19), (21) and (22) and substituting in equation (14), we can write the Carnot efficiency in terms of the Otto cycle parameters with the same extreme temperatures as

$$e_C = 1 - \frac{1}{r_{p_O} r_{V_O}^{\gamma-1}}. \tag{36}$$

Using the parameters of the Otto cycle depicted in Fig. 7 ($r_{V_O} = 10$, $r_{p_O} = 3$ and $\gamma = 1.4$), we find a Carnot efficiency of $e_C = 0.867$. This efficiency is greater than the Otto efficiency of $e_O = 0.602$, as was expected for this scenario.

To investigate how large the volume displacement of the Carnot cycle must be to attain the temperatures satisfying $T_{c_C}/T_{a_C} = T_{c_O}/T_{a_O}$, we substitute equation (36) on the work per cycle expressed by equation (16) and impose the argument inside the logarithm to be greater than one. This leads to the following condition

$$r_{V_C} > r_{V_O} r_{p_O}^{1/(\gamma-1)}. \tag{37}$$

This requires $r_{V_C} > 155.9$ when we want a Carnot Cycle that operates between the same extreme temperatures as that of the Otto cycle depicted in Fig. 7 ($r_{V_O} = 10$, $r_{p_O} = 3$ and $\gamma = 1.4$).

The compression ratio $r_{V_C} = 155.9$ and the efficiency $e_C = 0.867$ leads to a null work per Carnot cycle according to equation (16) since r_{V_C} must be greater than that value. Suppose now that the Carnot cycle is forced to perform the same work per cycle as the Otto cycle. In that case, it is possible to determine an expression for r_{V_C} by equaling the right sides of equations (16) and (28) with e_C given by equation (36). This leads to the relation

$$r_{V_C} = r_{V_O} r_{p_O}^{1/(\gamma-1)} \exp\left[\frac{(r_{p_O} - 1)(r_{V_O}^{\gamma-1} - 1)}{(\gamma - 1)(r_{p_O} r_{V_O}^{\gamma-1} - 1)}\right]. \tag{38}$$

In the case of using the parameters of the previous example ($r_{V_O} = 10$, $r_{p_O} = 3$, and $\gamma = 1.4$), we find $r_{V_C} = 495.6$ if we want a Carnot cycle with the efficiency $e_C = 0.867$ and work per cycle $W_C = W_O = 191.5\ J$. Due to the value of the maximum Carnot Cycle volume of $V_{c_C} = 12389.9\ mL$ significantly higher than $V_{c_O} = 250\ mL$ for the Otto cycle with the same extreme temperatures, both cycles can not be visualized in a p vs. V diagram with the same scale to be compared.

7. Conclusions

It is a well-known result that the Carnot cycle has the highest efficiency compared with any other thermodynamic cycle constrained to operate between the same high- and low-temperature thermal reservoirs. However, comparing the Carnot cycle and the Otto cycle constrained to operate with the same compression ratios allows us to conclude that the Otto cycle has higher efficiency and work per cycle at this configuration. Furthermore, we observe a non-monotonic dependence between the work per Carnot cycle and its efficiency. In our tests, even in the maximum work per Carnot cycle scenario, the work per Otto cycle seems higher for most pressure and compression ratios cases.

For a Carnot cycle with a compression ratio higher than that of the Otto cycle, we obtained the conditions for both to present the same work per cycle when constrained, separately, the same values for the efficiencies and the extreme temperatures ratio of both. The conditions we found indicate a Carnot compression ratio significantly higher than that of the Otto cycle. This means a theoretical Carnot engine with a piston stroke length much larger than an internal combustion engine with the same work per cycle. In addition, considering the displacement slowness inherent to the isothermal transformations, the theoretical Carnot engine would display fewer revolutions per minute compared to an internal combustion engine, meaning lower power even when performing the same work per cycle.

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